SYNOPSIS

Because of current trends in the design if rotating machinery towards higher running speeds, manufacturers are now tending to produce machines which operate closer to lateral critical speeds than has in the past been necessary. Consequently the effect of overhung couplings upon the critical speeds and vibration amplitudes of such machines is becoming an increasingly important consideration during coupling selection. Similarly, higher running speeds produce larger unbalance forces at couplings, which can in turn produce increased vibration amplitudes. The subject of coupling balancing is not well documented, particularly for couplings carried by flexible shafts, and coupling balancing standards tend to vary from organisation to organisation. Digital computers in conjunction with modern analysis techniques provide a means of assessing these effects, but this approach can be inconvenient at the initial coupling selection stage.

This paper, therefore, attempts to identify and quantify the main factors which influence the lateral vibrations of machines and to produce guidance for coupling selection based upon these factors. Two main areas are discussed:-

1. The ways in which coupling mass and the amount of overhang affect the critical speeds of two-bearing flexible rotors are quantified, and the results are presented in the form of a general guidance chart.

2. A method is suggested whereby existing, well established balancing criteria for rigid rotors might be adapted to provide balancing criteria for couplings carried by flexible rotors.

NOTATION

a, b, c	Dimensionless constants defined in Fig. 2(a).
d_1, d_2	Effective shaft diameters – see Appendix Tables A1 and A2,
e	Mass eccentricity.
e_c	Allowable mass eccentricity of coupling assembly.
e_f	Allowable mass eccentricity for a sub-critical flexible rotor.
e_m	Effective 'measured' mass eccentricity at balancing planes.
e_r	Allowable mass eccentricity for a rigid motor.
EI_1, EI_2	Effective shaft flexural rigidities – see Fig. 1 (2).
F_{sr}, F_{sf}	Forces experienced by the supports for a rigid rotor and a sub-critical flexible rotor, respectively.
F_{ur}, F_{uf}	Rotating central unbalance force for a rigid rotor and a flexible rotor, respectively.
L_1, L_2	Bearing centre distance and length of coupling overhang, respectively.
m_1, m_2	Effective central mass of rotor and effective coupling mass, respectively.
m_s, m_t, m_o, m_c	Masses required for critical speed calculations, defined in Appendix Table A1.
N	Normal operating speed of rotor (rev.min).
N_b	Rotational speed at which rotor is balanced (rev/min).
N_c	Lateral critical speed of rotor assembly (rev/min).
N_{co}	Lateral critical speed of rotor alone, as computed by conventional methods or based on manufacturers information (rev/min).
T	Support transmissibility = $1/[1-(N/N_c)^2]$

INTRODUCTION

The main functions of a flexible coupling are to transmit torque and to allow misalignment and, in the past, it has often been adequate to select a coupling purely on the basis of these two factors. However, with the current trend towards higher torques and rotational speeds it is becoming increasingly apparent that vibration characteristics can place a considerable limitation upon machine performance, and that flexible couplings can play a detrimental role in this respect. Digital computers in conjunction with modern analysis techniques provide a means of assessing the vibration characteristics of coupled machines, often with quite a high degree of accuracy. However, at the initial coupling selection stage this approach can be inconvenient, even if a computer program is available. Moreover, computer analyses, by their very nature, are aimed at particular applications and do little to assist the designer in formulating a simple, general coupling selection procedure.

Unfortunately, general theoretical studies of the effects of couplings upon machine vibrations have been few, and added to this is the fact that the physical factors which limit the acceptable vibration levels of a rotating machine are not clearly understood. Consequently the available information on the subject of machine vibrations and the effects of flexible couplings upon these vibrations, tends to range from highly mathematical theoretical studies to a variety of experience-based standards, with remarkably little connecting these two extremes. This paper is therefore intended to fill this gap and, hopefully, to provide the designer with sufficient information to allow him to select a coupling for a particular application without the needing to resort to digital computations, at least for the initial selection stage.

The effects of an overhung flexible coupling upon the vibrations of coupled machines can be separated into two categories:-

(i) The mass, geometry and location of the coupling affect the lateral critical speeds of the system, which in turn affect the vibration levels.

(ii) The vibration levels are also directly influenced by the amount of coupling unbalance.

These are therefore the obvious areas to consider when attempting to formulate coupling selection methods, and in this paper they are treated in the following ways:-

(i) The effects of an overhung coupling upon the critical speeds of a machine are examined by using a model which, although simple, takes into account not only the mass and position of the coupling, but also the rotor mass, the shaft stiffness between the bearings, and the stiffness of the overhung section of shaft. A simple, approximate method for checking the first critical speed of machines with overhung couplings is then presented, and some general conclusions about the desirable features of such a system are made.

(ii) Coupling balancing requirements are investigated by examining several existing machine vibration and balancing standards in an attempt to identify a fundamental factor which limits the level of vibrations which are acceptable. Based on this limiting factor, the investigation goes on to show how existing, well established balancing criteria for ‘rigid’ rotors could be modified for ‘flexible’ rotors operating below their fundamental flexural critical speeds, and the arguments are then extended to cover the balancing of couplings. By using simple methods of analysis throughout the salient factors which affect coupling balance requirements are not obscured by mathematical complexity.

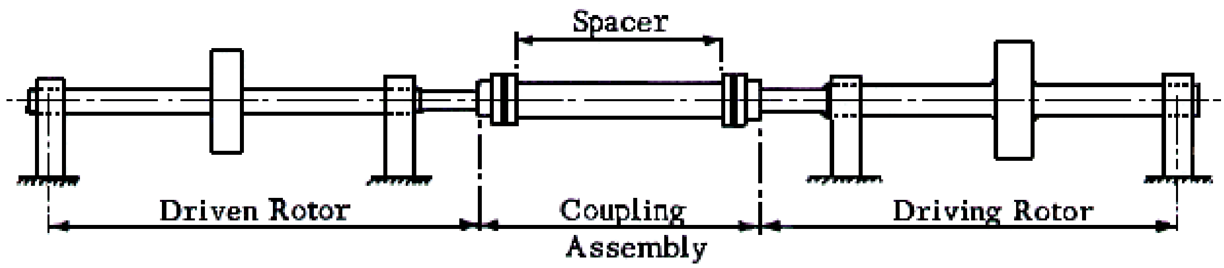
The terminology used in this subject area is a little confused and deserves some discussion. Some machine manufacturers use the term ‘flexible’ to describe a shaft operating at a speed greater than its first critical speed. Similarly, they describe a shaft running at a speed below its first critical as ‘stiff’. The terminology used throughout this paper is the classical terminology in which a ‘stiff’ or ‘rigid’ shaft is one which does not deflect under the action of vibrational excitation forces. Obviously no such shaft can exist but a shaft running at a speed of less than about a quarter of its first critical speed approximates to a ‘rigid’ shaft. Thus, according to the terminology used in this paper, all real shafts are ‘flexible’. The terms ‘sub-critical’ and ‘super-critical’ are then used t denote a shaft running either below or above its first critical speed, respectively. This terminology has been used throughout because it is believed to convey a more accurate impression of the true situation.

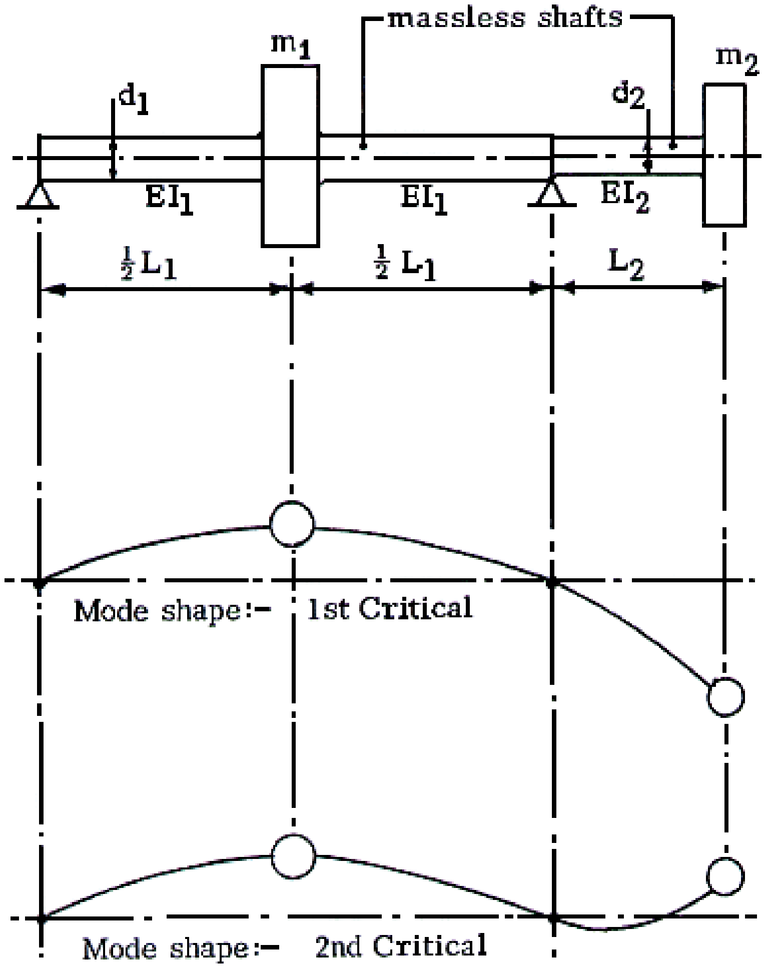
THE EFFECTS OF COUPLINGS UPON LATERAL CRITICAL SPEEDS

Fig. 1 shows a simple model of a pair of two-bearing rotors connected by a spacer-type flexible coupling. It can be shown that the degree to which the vibrations of one rotor affect those of the other rotor is related to the mass m_s of the spacer, so that a massless spacer would totally prevent any such effect. In practice most modern couplings incorporate spacers which are relatively light in comparison with the weight at each of the points of articulation. It is therefore reasonable to assume that the two rotors vibrate independently, and to consider each as a separate rotor carrying half of the total mass of the coupling/ spacer assembly, as shown in Fig. 2(a). This simple model treats each rotor as a two mass system, which therefore has two critical speeds. The mode shapes corresponding to each of these criticals are illustrated in the figure, and for the majority of applications it is the first critical, being the lowest of these speeds, which will be important from the point of view of rotor vibrations in service. This first critical speed corresponds to the fundamental critical speed of the rotor, modified to a lower value by the mass of the overhung coupling.

A method of using this model in order to calculate approximately the amount by which the coupling reduces the fundamental critical speed of the rotor, is given in the Appendix. This method involves relatively simple formulae and has the important feature that it uses the fundamental critical speed of the rotor alone (i.e. without a coupling mass) as calculated by conventional design methods in order to determine an equivalent shaft diameter between the bearings. It can therefore take into account shaft stiffening effects from seals or other such features.

For general design guidance it is also useful to have some indication as to which of the various features of the machine and coupling design are likely to have the greatest effect on the critical speed, and this is given in Fig. 2 (b) which is based on results calculated using the methods described in the Appendix, and gives an indication of the reduction in critical speed due to the various features. Shaft overhang has a greater effect than coupling mass, but quite small coupling masses can have a large effect if the overhung portion of the shaft is of relatively small diameter. Fig. 2 (b) suggests that, ideally, the rotor shaft and coupling should always be selected so that ‘a’ is less than 0.1. Where this cannot be achieved and higher values of ‘a’ have to be accepted, the coupling mass becomes progressively more critical. It can also be seen from Fig. 2 (b) that it is essential to keep values of ‘c’ s high as possible, and preferably greater than about 0.1.





Dimensionless Parameters

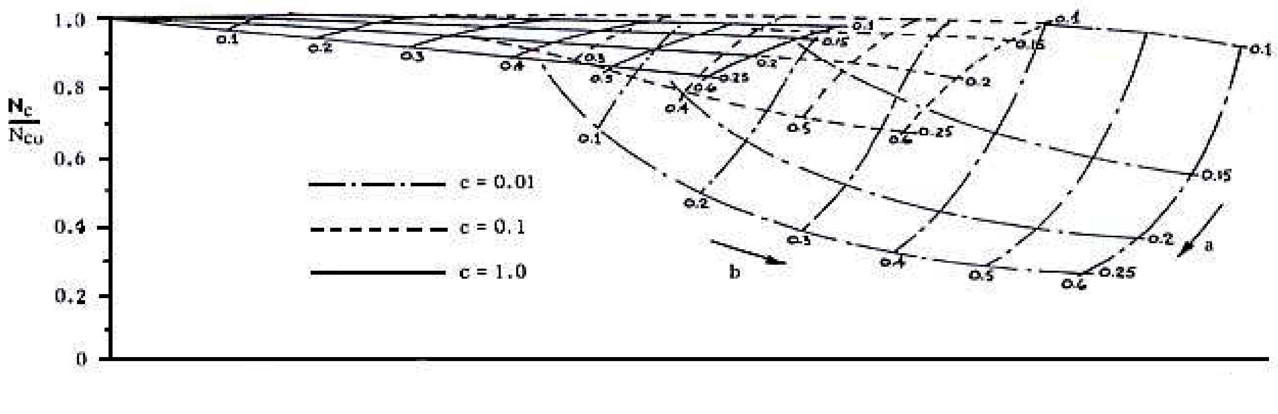
$a = L_2/L_1$

$b = m_2/m_1$

$c = EI_2/EI_1$

(a) Simplified Model of a Rotor with an Overhung Coupling.

N_C = Critical speed with coupling N_{CO} = Critical speed without coupling



(b) An Indication of the Reduction in Critical Speed due to the Fitting of a Coupling.

FIG. 2

THE REQUIRED STANDARD OF COUPLING BALANCE

For high speed shafts, the rotating forces generated by a flexible coupling due to unbalance can be quite large, particularly at the top end of the speed range, because the centrifugal forces due to unbalance increase as the square of the rotational speed. It is therefore possible for high levels of vibration and high bearing forces to exist even for a rotor which is operating with a safe margin on its critical speed. As a general rule the vibration levels and bearing forces increase as the critical speed is approached, and the rate of increase starts to become rapid in the region of about 0.8 of the first critical speed. It is not uncommon for modern high speed machines to operate in this region, and in order to achieve an acceptably low level of vibration, careful attention must be paid, when selecting a coupling, to the amount of coupling unbalance which can be tolerated, and also to whether or not the desired level of balance can be achieved for a particular coupling design.

There are several balancing standards available for rotating machines, but none of these appears to take into account the ratio of running speed to first critical speed. Consequently, the designer faced with the task of selecting a coupling for flexible-rotor machinery, specifying its allowable unbalance and also specifying the method of balancing, has only a very limited amount of information to work from. This section of the paper attempts to provide the necessary information by developing a possible method for assessing coupling balance requirements which takes into account not only machine class, but also the ratio of running speed of the machinery to its first critical speed (sub-critical machines only). Four main steps are involved:-

(1) Existing machine balancing and vibration standards are examined in an attempt to identify a possible key factor which limits the allowable vibration levels of machines in service. It is found that the dynamic forces experienced by the bearings appears to be such a factor.

(2) The possibility that this factor can be used to define machine balancing requirements is verified by developing a rigid-rotor balancing criterion from an existing machine vibration standard, using additional theoretical and practical data.

(3) On the basis of these findings, a method of modifying existing, well established balancing standards for rigid rotor machines to include rotor flexibility effects is suggested.

(4) The arguments are then extended to suggest balancing standards for couplings carried by sub-critical flexible rotors.

The resulting method is summarised later in the paper, but in order to provide the designer with the necessary 'feel' for the subject, the above steps are discussed in greater detail in the following sections.

Some Existing Balancing and Vibration Standards

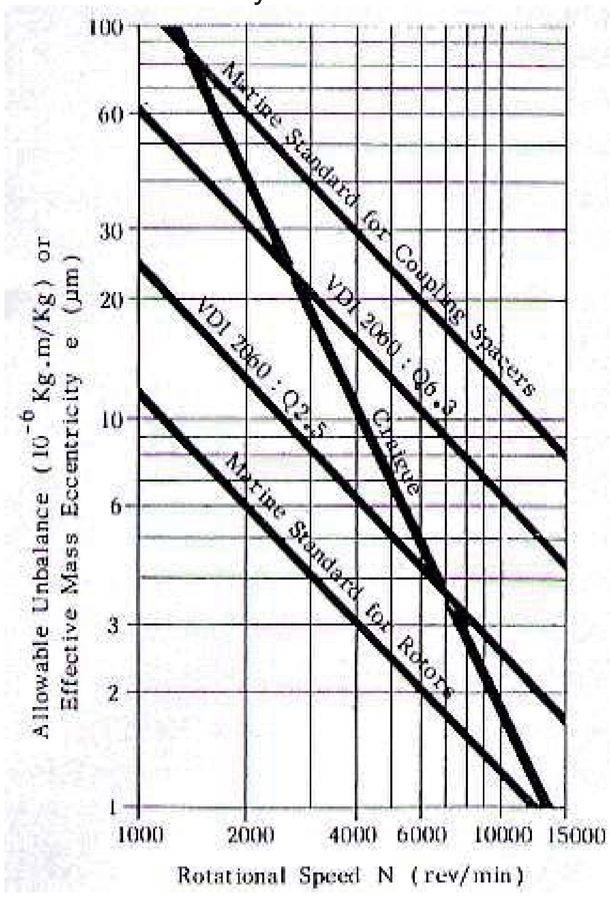
A logical method of assessing the allowable degree of unbalance is to relate it to the levels of vibration which are known to be acceptable. There are various methods of assessing allowable vibration levels, but these are usually based on measurements such as the vibration velocity of the bearing housings, which need to be related to the amount of unbalance if they are to provide the basis for the development of balancing criteria. In order to do this, some of the existing balancing and machine vibration level standards have been examined in order to determine a fundamental factor which links them. The various standards considered are outlined below:-

1. Balancing Standards:-

Fig. 3 shows a comparison of three types of balancing standard which are in common use. One of the better known is VDI 2060 (Ref.1), in which the allowable mass eccentricity 'e' of the rotor is related to the operating speed 'N' for various classes of machine. However, this code of practice is intended for 'rigid' rotor machines only. A similar standard for flexible rotors was to be produced at a later date but this, to the author's knowledge, has not yet appeared. One of the most interesting features of VDI 2060 is the general form of the criteria, which is :-

$$e.N = \text{a constant} \qquad (1)$$

This linear relationship is explained in VDI 2060 as being based upon 'statistical evidence' and 'practical experience'. Of the ten machine classes in VDI 2060, quality classes Q6.3 and Q2.5 are probably the most appropriate for the type of high speed machinery of interest at this conference but, as has already been stated, apply to rigid rotor machines only.



A Comparison of Balancing Standards

FIG. 3

N.F. Craigue (Ref.2.) of the Van Karman Gas Dynamics Facility, Tennessee, describes their rotor balancing criteria thus: “If the centrifugal force created by an unbalance weight exceeds 20% of the weight of the rotating assembly, the resulting vibrations will be excessive”. No further explanation is given, but it is likely that this statement is based upon practical experience gained with high speed compressors. This criterion effectively relates the allowable mass eccentricity to the ‘square’ of the rotor speed. Rothfuss (Ref.3.) points out that this type of criterion is gaining popularity in the U.S.A.

Marine installations are commonly balanced to B.S.3853: 1966 (ref.4) – often referred to as the ‘marine standard’ – which takes the form: -

$$\text{Allowable Unbalance (oz.in)/balancing plane} = \frac{K \times \text{Weight (lb)}}{\text{Shaft Speed (rev/min)}} \qquad (2)$$

where K = 4 for turbine rotor assemblies (including the coupling hub but excluding the spacer) and K = 40 for the coupling spacer. This standard can also be expressed in terms of an effective mass eccentricity, as shown in Fig. 3.

Bearing in mind that these three types of balancing standard are compared on a logarithmic scale in Fig. 3, the amount of variation between them can be seen to be quite large. It is therefore extremely difficult to know which to use for a particular application.

2. Vibration Standards:-

A commonly used code of practice for assessing the acceptability of machine vibration levels in service is VDI 2056 (Ref.5), which utilizes the vibration velocity of the bearing housings of the machine as the yardstick, since this is considered to reflect the severity of the vibrations and the resulting fatigue stresses. The assessment charts given in that standard are based partly upon theoretical considerations, but mainly upon a great deal of user experience. This code of practice is extremely useful for assessing machines already in service, but it has the disadvantage that it is very difficult to use at the design stage when attempting to determine the acceptable degree of rotor unbalance, because to do so would require a knowledge of the dynamic stiffness and effective mass of each bearing pedestal.

Another vibration criterion has been given by Erskine (Ref.6). This applies to rotors running in plain journal bearings, and uses the vibration amplitude of the journal relative to the bearing clearance as the basis for vibration assessment (see Table 1). Erskine states that journal amplitudes much greater than those given in Table 1 represent abnormal operation. These values are based upon experience rather than upon theoretical considerations, and it is likely that they apply to both stiff and flexible rotor machines.

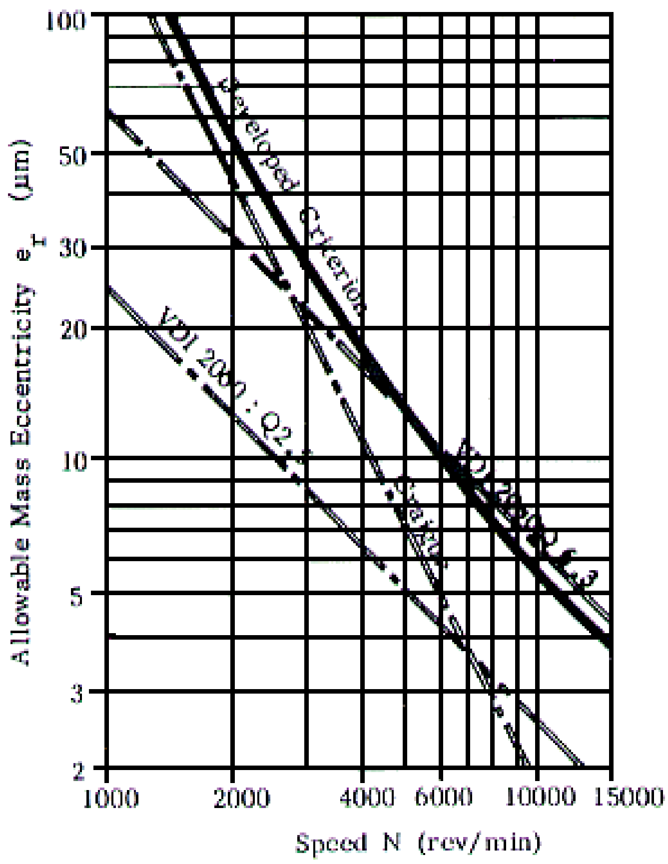
Table 1: Erkine's Vibration Criterion

Shaft Speed N rev/min.	Journal Amplitude / Radial Clearance of Bearing
300	0.50
3000	0.25
12000	0.10

Although no obvious simple relationship exists between these two vibration standards both are based upon vibration levels measured at the bearings. Since these vibration levels are strongly dependent upon the rotating forces transmitted to the bearings, it seems reasonable to conclude that it is these transmitted forces which are the key to defining the maximum allowable vibration levels for satisfactory operation of the machine.

The Development of a Rigid-Rotor Balancing Criterion

If, as is suggested in the previous section, the rotating forces experienced by the bearings is a key factor in defining acceptable machine vibration levels, then it follows that, by relating these forces to the amount of unbalance, it should be possible to develop rotor balancing standards from existing machine vibration standards. Moreover, because all of the various vibration and balancing standards outlined previously in this paper are based largely upon user experience, any balancing criterion developed in this way should show a reasonable agreement with existing balancing standards. The author has, in fact, tested this possibility using a combination of theoretical and empirical data in conjunction with Erskine's vibration criterion. The way in which this was done is described in detail in a previous paper (Ref. 7), and the resulting balancing criterion is compared with existing standards in Fig. 4 below.



Developed Rigid-Rotor Balancing Standard.

FIG. 4

Even though the methods used to develop this criterion were very approximate the agreement with VDI 2060 balancing class Q6.3 at speeds above about 3000 rev/min and also with Craigue's 'speed-squared' criterion below 3000 rev/min, is interesting. A good deal of the experience upon which Erskine's vibration-amplitude criterion is based was gained in the petro-chemical industry where machines of VDI balancing classes Q6.3 and Q2.5 are common, and this fact makes Fig. 4 even more remarkable.

The developed criterion was not intended to replace existing balancing standards. Instead, it serves to illustrate the point that the rotating forces transmitted to the bearings do appear to provide a suitable basis for the development of simple flexible-rotor balancing criteria on the basis of this conclusion and using an existing rigid-rotor balancing standard rather than the developed one in Fig. 4, since the former is well established and takes into account the machine type.

Possible Balancing Standards for Sub-Critical Flexible Rotors

For a rigid symmetrical rotor with unbalance at the mid-bearing position the rotating force experienced by each bearing is equal to half of the rotating unbalance force, as illustrated in Fig. 5(a).

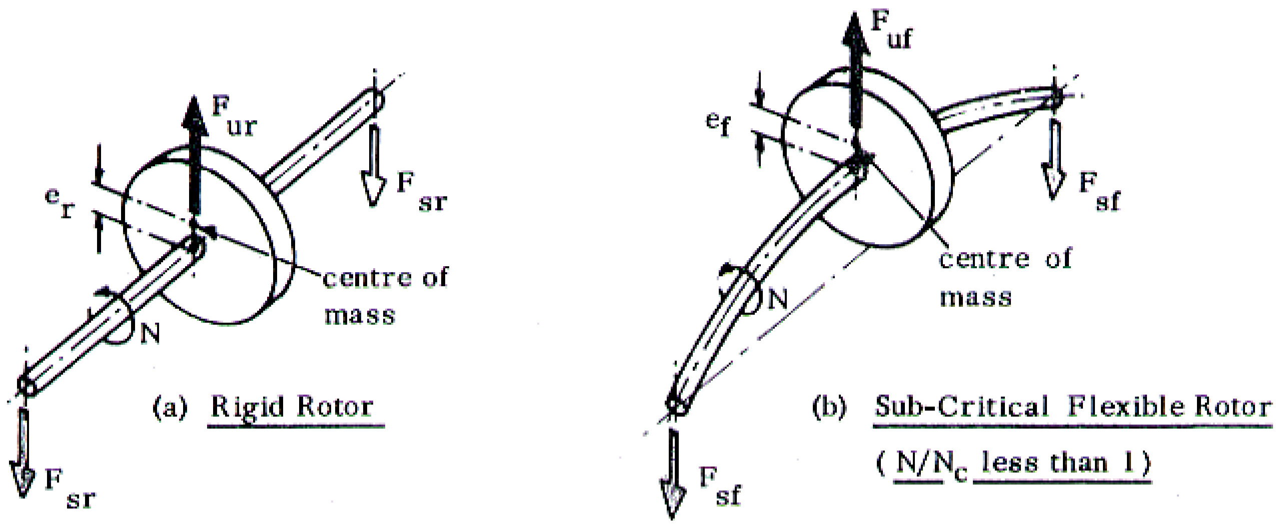


FIG. 5

If, on the other hand, the shaft is flexible as shown in Fig. 5(b), and the first critical speed of the rotor assembly if N_c, the forces experienced by the bearings at a running speed N will be greater than half the rotating unbalance force by a factor T known as the support transmissibility, where :-

$$T = \frac{1}{1-\left(N/N_c\right)^2} \qquad (3)$$

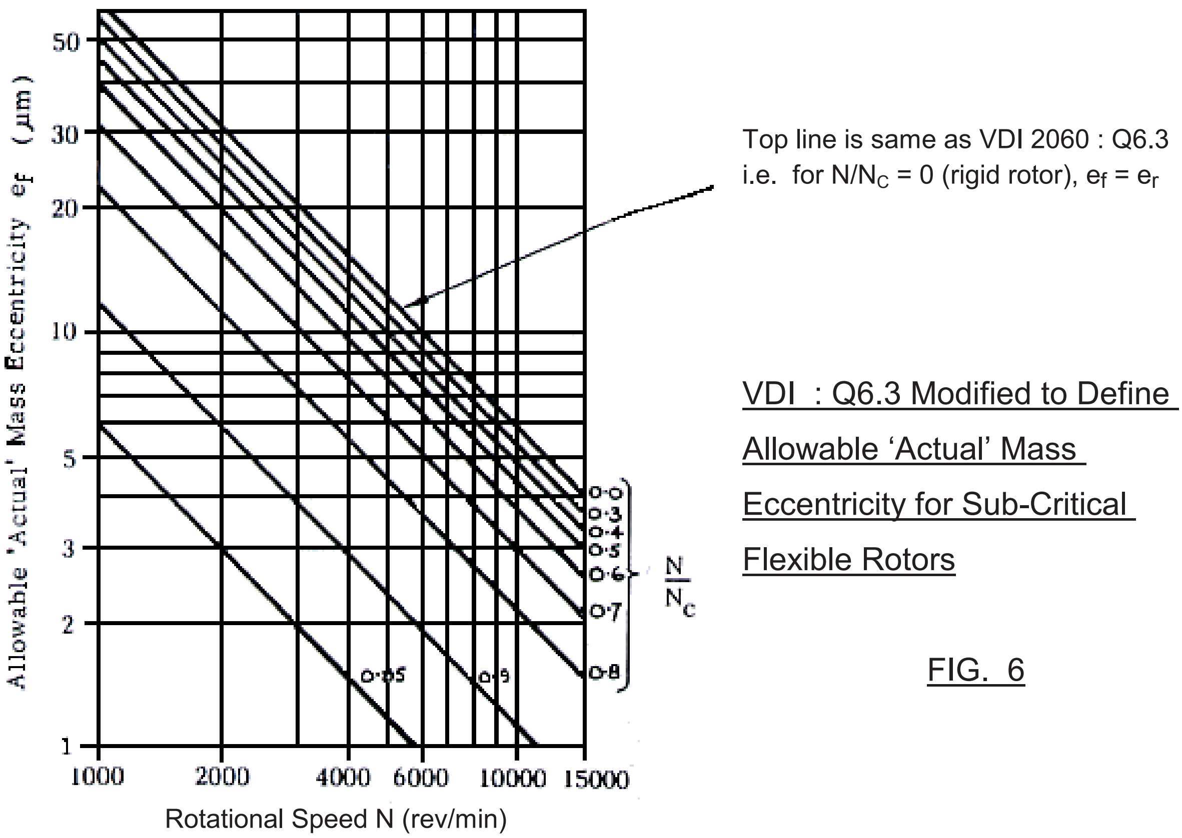
Therefore, in order to ensure that the forces experienced by the bearings for a flexible rotor (F_{sf}) do not exceed those for the equivalent rigid rotor (F_{sr}), since these forces appear to limit machine performance, it is necessary to reduce the central unbalance force (F_{uf}) to a value which is lower than that considered acceptable for the rigid rotor (F_{ur}) in the following way:-

$$F_{uf} = F_{ur}\left[1-\left(N/N_C\right)^2\right] \qquad (4)$$

or, in terms of the maximum allowable eccentricity of the central mass :-

$$e_f = e_r\left[1-\left(N/N_C\right)^2\right] \qquad (5)$$

To illustrate the significance of this equating, consider a machine which, if it had a rigid rotor, would correspond to VDI 2060 balancing class Q6.3. When this is modified to define the allowable mass-eccentricity for a sub-critical flexible rotor, using equation (5) above, the new criterion takes the form shown on Fig.6.



VDI : Q6.3 Modified to Define Allowable 'Actual' Mass Eccentricity for Sub-Critical Flexible Rotors

FIG. 6

As would be expected, the required degree of balance increases markedly as N/N_c approaches unity, but is not greatly different from the rigid-rotor criterion for N/N_c values of less than about 0.3. Equation (5) applies to undamped rotors only, but should the rotor have some mid-support damping, the simple single-degree-of-freedom model is easily modified to include this effect. However, the amount of damping for a particular application is not always easily defined, and in this situation it is probably wise, from the point of view of balancing requirements, to assume zero damping.

Before extending the arguments to include coupling balancing, it is important to point out exactly what equation (5) and Fig. 6 mean. When balancing a rotor assembly it is normal to measure the dynamic forces at measuring planes close to the bearings. Consequently, for a flexible rotor with the unbalance situated at the mid-support position, the dynamic forces measured at the measurement planes will not be equal to the central unbalance force, but will be magnified by the support transmissibility T. If the rotor is balanced at its proposed operating speed, then the transmissibility effect will be completely included in the measurements. If the force measured at the bearings is converted to an effective 'measured' mass eccentricity e_m by dividing the force by the product of the rotor mass and the square of the rotational angular velocity, then the 'measured' mass-eccentricity will be greater than the actual mass-eccentricity by a factor equal to the inverse of the transmissibility. Hence, the measured eccentricity need not be reduced by balancing to a value less than that considered acceptable for the same class of rigid rotors:-

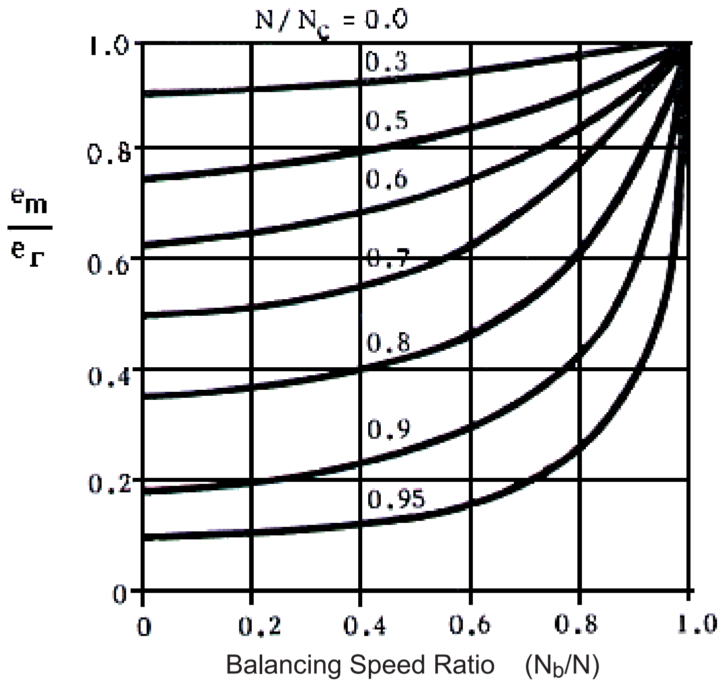
i.e. $e_m = e_r$ (= e_f for N/N_c = 0.0 in equation 5)
(6)

Should such a flexible rotor be balanced at a speed N_b which is less than the operating speed N, then the transmissibility effect will be only partly included in the measurements, and for this situation the maximum allowable 'measured' mass eccentricity is given by:-

$$e_m = e_r.\left[\frac{1-(N/N_c)^2}{1-(N_b/N_c)^2}\right]$$

(7)

This relationship is shown in Fig. 7.



Variation of Allowable 'Measured' Mass Eccentricity with Balancing Speed Ratio.

FIG. 7

Now, for any flexible rotor which cannot be balanced as an assembly, or which for some reason has to be dismantled for installation, then in order to ensure that the forces experienced by the bearings do not exceed those for the equivalent rigid rotor, the maximum allowable 'actual' mass-eccentricity of the rotor components must not exceed those given by the modified balancing standard (such as that shown in Fig. 6) for the appropriate N/N_c ratio.

To illustrate these various points, consider an idealized sub-critical flexible rotor such as that given previously in Fig. 5(b), and assume that this is to be balanced in such a way that the forces experienced by the bearings due to unbalance are not greater than those for the equivalent rigid rotor balanced to VDI 2060, Q6.3. For the purpose of this example assume that the normal rotor running speed is 3,000 rpm, and that this is 80% of the first critical speed (i.e. $N/N_c = 0.8$). If this rotor were to be balanced on a two-plane balancing machine at its normal running speed with the balancing planes close to the bearing positions, then the maximum allowable 'measured' mass-eccentricity is given by Fig. 6 as 20 μm, i.e. the same as for the equivalent rigid rotor (e_r). If, on the other hand, the balancing speed id only 1500 rpm (i.e. $N_b/N = 0.5$) then, using Fig.7, the measured mass eccentricity must not exceed 0.44 e_r, i.e. about 9 μm. Finally, if the central mass were to be balanced separately on a rigid mandrel, then the maximum allowable mass eccentricity is given by Fig. 6 for $N/N_c = 0.8$ as about 7.5 μm.

Allowable Coupling Unbalance

The final stage is to extend these arguments to include the effects of coupling unbalance. This is the most difficult step because the dynamics of a flexible rotor with both a central mass and an overhung coupling mass are rather more complex, in terms of the rotating forces experienced by the bearings due to the combination of rotor and coupling unbalance, than for the simple, single mass model considered so far. These bearing forces are dependent not only upon the geometry of the rotor / coupling assembly and the ration of running speed to critical speed, but also upon the angular displacement between the rotor unbalance and that of the coupling.

As a first approximation, however, the balancing requirements for an overhung coupling can be taken to be the same as for any mid-bearing rotor-mounted mass, as described in the previous section. In other words, the allowable mass-eccentricity e_c for a mandrel balanced coupling is given by:-

$$e_c = e_r \cdot \left[1 - (N/N_c)^2\right] \qquad (8)$$

where N_c is the first critical speed of the rotor / coupling assembly, and e_r is the allowable mass-eccentricity of the rotor if it were rigid. In fact, the coupling will be carried by two machines, and therefore equation (8) should be used for each machine in turn, and the lower value should be used to define e_c.

Balancing Requirements : Summary of Findings

(1) The magnitude of the dynamic forces experienced by the bearing of a rotating machine appears to be a key factor in defining all allowable vibration level for that machine.

(2) For a perfectly rigid rotor, the sum of the forces experienced by the bearings is equal to the total unbalance force.

(3) For a simple, two-bearing flexible rotor operating below its first critical speed, as illustrated earlier in Fig. 5(b), the sum of the bearing forces is greater than the unbalance force by a factor T, which is defined in equation (3).

(4) Balancing criteria for sub-critical flexible rotors and their components are suggested, which are based upon (1), (2) and (3) above and which utilize existing, well established rigid-rotor balancing standards. The examples given are based upon VDI 2060, but the method can be equally well applied to other rigid-rotor balancing standards.

(5) When balancing a sub-critical flexible rotor / coupling assembly at its normal operating speed, and with balancing planes close to the bearing positions, the maximum effective 'measured' mass eccentricity of the rotor e_m should not exceed that for the equivalent rigid-rotor machine e_r. In the examples given e_r is the value defined by VDI 2060.

(6) When balancing such an assembly at a speed lower than its normal operating speed, the effective 'measured' mass eccentricity e_m should not exceed the value given by equation (7) which will always be less than the allowable mass eccentricity for the equivalent rigid-rotor machine.

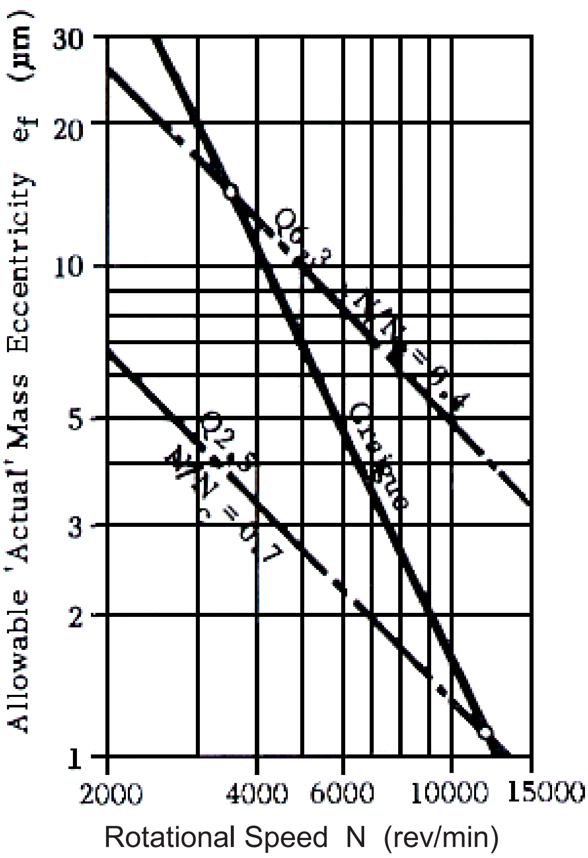
(7) If a coupling which is to connect two sub-critical flexible rotor machines is separately balanced on a mandrel, then the allowable mass eccentricity is given by equation (8). The value is always less than e_r which is the lower of the allowable mass-eccentricities for the two rotors given by the appropriate 'rigid rotor' balancing standard.

An Explanation of Existing Balancing Standards

In order to complete the picture, it is useful to attempt to explain the differences between some of the existing balancing standards, and this is done below, using VDI 2060 modified for rotor flexibility as explained in the previous sections.

As mentioned earlier in this paper, 'speed-squared' balancing criteria of the type described by Craigue (see Fig.3) seem to be gaining popularity. This type of balancing standard takes a very different form to all of the others mentioned in this paper. However, the methods given in this study can be used to give a possible explanation for this difference. It can be argued that, because of mechanical and tribological limitations, the designer of high speed rotors is often forced to accept greater rotor flexibility than for lower speed machines. It is therefore likely that, for a particular class of machines, some sort of relationship will exist between rotor speed N and the criteria speed ration N/N_C, and that the higher N/N_C values will correspond to the higher values of N. Thus, a balancing criteria based upon experience with rotors of varying flexibility would be expected to reflect such a relationship.

Craigue states in his paper (Ref.2) that the speed range of the compressors for which this balancing criterion is used is about 3,500 – 12,000 rev/min. If these were rigid rotor machines it is likely that the appropriate VDI 2060 balancing classes would be Q6.3 at the lower end of this speed range, and Q2.5 at the higher end. When these two VDI classes are modified for rotor flexibility and compared with Craigue's criterion, as shown in Fig.8, it can be seen that they are in agreement with Craigue's criterion at 3,500 rev/min for $N/N_C = 0.4$, and at 12,000 rev/min for $N/N_C = 0.7$. Although critical-speed ratios are not quoted by Craigue, these values do not appear to be too unrealistic. This implies that speed-squared criteria do include rotor flexibility effects, and could be gaining popularity for this reason.



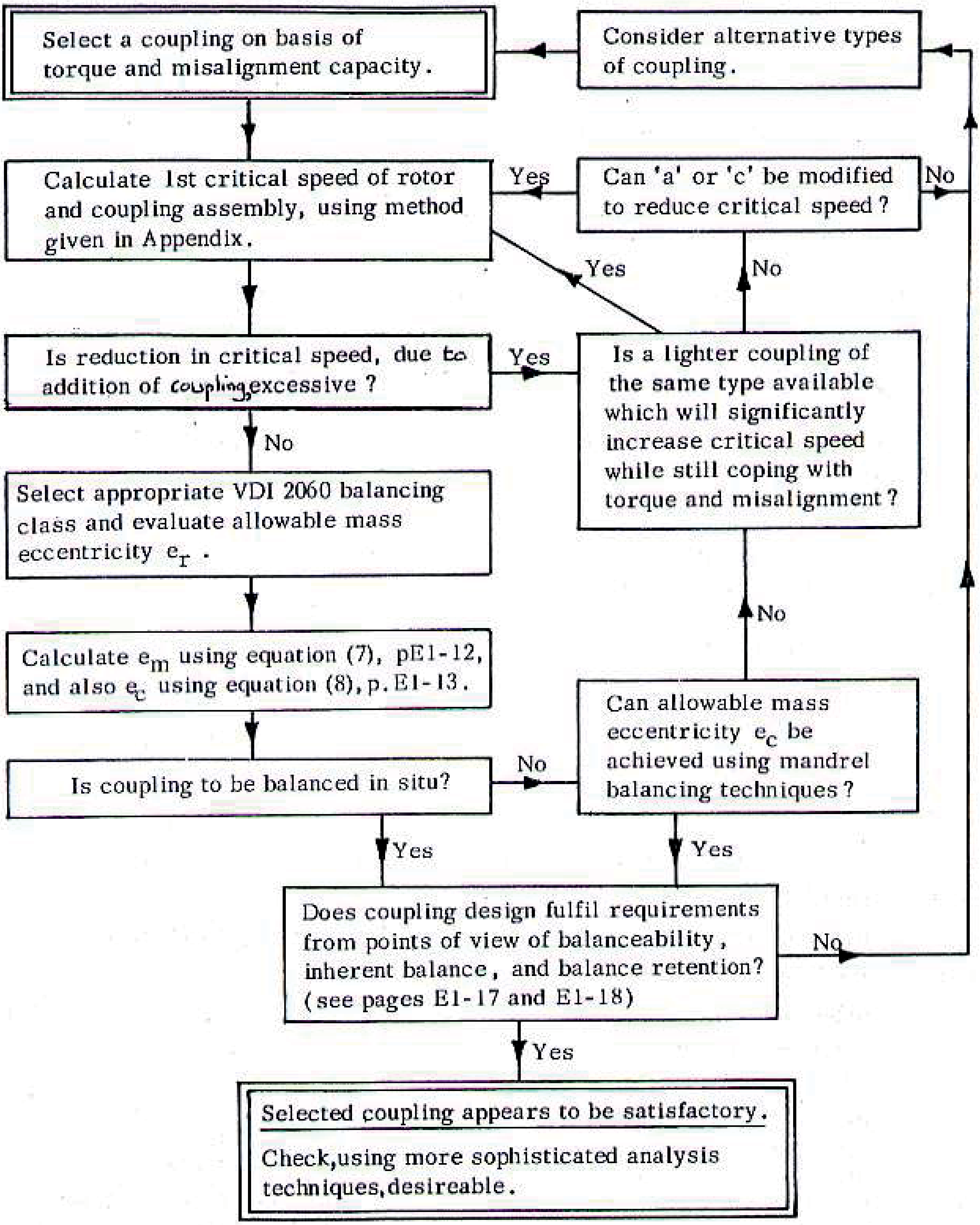
<u>Comparison of VDI 2020 (Modified) and Craigue's Criterion</u>

<u>FIG. 8</u>

It is difficult to explain how the marine standard BS 3853:1966 came about. The appropriate VDI 2060 class for a turbine, if the rotor were rigid, would be Q2.5 and, if balanced at its operating speed with balancing planes close to the bearings, the acceptable value of the 'measured' mass-eccentricity for a flexible rotor is the same as for the equivalent rigid rotor, which in this case is that given by VDI 2060: Q2.5. This implies that the marine criterion gives an unduly high standard of rotor balance, as shown previously in Fig.3. However, the addition of a coupling spacer balanced according to BS 3853:1966, to a much lower standard than the rotor, would increase the overall mass-eccentricity of the system. The amount by which it increases would depend upon the relative weights of the rotor and the spacer. Rothfuss (Ref.3) observes that the marine standard was developed at a time when very heavy flexible couplings were being used and, this being the case, it is possible that the resulting overall mass-eccentricity for a system balanced in this way could have been similar to that given by VDI 2060:Q2.5.

DISCUSSION

The various methods and relationships developed in this paper cannot be expected to provide the designer with highly accurate data for calculating the effects of couplings upon machine vibrations. Several parameters which can be important (e.g. journal bearing lubricant-film flexibility, bearing pedestal stiffnesses, rotor mass distribution, external damping etc.) have been neglected for the sake of mathematical simplicity. Nevertheless, if applied carefully the suggested methods should provide the designer with sufficient data for estimating coupling effects at the preliminary selection stage. To this end, Fig.9 shows a suggested flow diagram for coupling selection on the basis of machine vibrations which uses a modified version of VDI 2060 to define the coupling balance requirements.



Suggested Flow Chart for Preliminary Coupling Selection Stage Based on Vibration Considerations and Using VDI 2060.

FIG. 9

In the first part of the paper, the way in which the first lateral critical speed of a machine is reduced by the addition of an overhung coupling, has been described, and referring again to Fig. 2(b) it can be seen that, depending upon the overhung configuration and coupling mass, this reduction can be quite significant. It has also been shown how the allowable coupling unbalance decreases as the ratio of running speed to first critical speed (N/N_C) increases. Therefore, as a general rule, for coupling selection it can be concluded that for a given overhang configuration, the higher the coupling mass the lower the first critical speed of the machine, and consequently the lower the allowable mass eccentricity of the coupling. The advantage of keeping coupling weight down are therefore obvious, particularly for vibrationally sensitive machines (i.e. machines operating at high N/N_C values). Similarly, it is advantageous to minimise the overhang length. Some types of coupling can be supplied with 'reversed' hubs (i.e. with the point of articulation close to the bearing end of the hub rather than the spacer end), and this helps to reduce the distance between the bearing and the point of application of the spacer mass, and consequently reduces the effective overhang length. The third factor which effects the first critical speed is the flexural rigidity of the overhung shaft, and wherever possible significant reductions in shaft diameter at the overhang should be avoided. Recommendations on limiting values for these various parameters are given earlier in this paper.

Referring again to Fig. 8, it can be seen that for high speed, high N/N_C machines, coupling mass eccentricities as low as 1 or 2 μm may be required. For this situation, in-situ coupling balancing is strongly recommended since, even if this degree of balance can be achieved using mandrel balancing techniques, it is likely that this state of balance will deteriorate when the coupling is installed on the machines, because of assembly tolerances.

Other features which deserve careful consideration when selecting couplings for vibrationally sensitive machines are:-

(1) Balanceability.
(2) Inherent balance.
(3) Balance retention.

Balanceability depends largely upon the type of coupling, and is of importance if the coupling cannot be balanced in-situ. Gear couplings require the application of torque in order to centre the sleeve and spacer relative to the pitch-circle-diameter of the hub teeth and consequently, due to gear teeth clearances, a gear coupling balanced as an assembly in the 'zero-torque' condition need not be correctly balanced when installed and operating under full torque. For this situation the balanceability is poor. The problem is not, of course, insurmountable. Tooth-tip centring has been used in an attempt to circumvent this effect, but it could be argued that when the coupling is operating at high speed, and under full torque, the torque centring action may take over from the tip-centring and, again, the operational balance could be inadequate. Multiple membrane couplings can suffer from a similar problem because of membrane distortion under torque, which could cause shifting of the spacer centre relative to the hub. Compensation balancing is a possible solution to this type of problem, and this involves balancing the coupling to a higher standard than that required by an amount equal to the possible increase in unbalance when under operating torque. Another possibility which may be worth investigation is the use of 'locked-in' torque during balancing, by means of torsion-bar balancing mandrels.

Inherent balance is very much dependent upon coupling design features and dimensional tolerancing, and is again of importance if the coupling cannot be balanced in-situ. A 'perfect' coupling in this respect is one which can be stripped and reassembled without a deterioration inbalance.

Balance retention is largely governed by mechanical integrity. A coupling in which components can shaft relative to each other during service, can deteriorate in balance (a poor clamping arrangement for membrane packs in multiple membrane couplings is a possible example).

Finally, it is worth noting that situations are not uncommon in which a rotor/coupling assembly, initially balanced to some well established standard, has been found to have deteriorated significantly in balance during service, without any noticeable deterioration in vibration performance. This suggests that some variation exists in balance requirements for machines of broadly the same type. The rotor flexibility effects described earlier in this report could partly explain this variation, and other factors inherent in the machine design will also contribute. Because of this, experience-based balancing criteria such as VDI 2060 will continue to play an important role, but it is tentatively suggested that they could be augmented by the type of methods presented in this paper, in order to develop coupling balancing standards for particular machines rather than just machine classes.

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APPENDIX

APPROXIMATE METHOD FOR ESTIMATING LATERAL CRITICAL SPEEDS OF ROTOR AND COUPLING ASSEMBLIES

The following method can be used to estimate the lateral critical speeds of a pair of sub-critical, flexible-rotor machines, each supported in two bearings and connected by a spacer type coupling, as illustrated by the simple model given in Fig. 1 in the main text of this paper. Each machine should be considered separately on the assumption that its vibration characteristics are not dependent upon those of the other machine.

Table A1 lists the various items of machine and coupling data which are required. The actual calculations are listed in Table A2 in their correct sequence. It is essential that the units specified are adhered to, because some of the equation constants given in Table A2 are based upon this system of units.

Table A1: Data Required for Critical Speed Calculations

Symbol	Units	Description
d_2	metres	Average diameter of overhung section of rotor shaft.
L_1	metres	Bearing centre span.
L_2	metres	Effective overhang length of coupling, measured between the bearing centre and the 'effective' centre of gravity of the coupling (see note below).
m_s	Kg	Distributed mass of rotor between bearings, excluding large, centrally located masses, such as impellers, etc.
m_t	Kg	Sum of centrally located, large, mid-bearing masses.
m_o	Kg	Mass of overhung section of rotor shaft, excluding coupling mass.
m_c	Kg	Effective rotor-carried mass, i.e. mass of coupling up to the mid-length position of spacer (this is not necessarily equal to half of the total coupling mass).

Note: Defining the coupling end-mass as that part of the coupling carried by the shaft end up to the point of articulation, then the 'effective' C. of G. of the coupling will lie between the C. of G. of this end-mass and the point of articulation.

Table A2: Critical Speed Calculations

Symbol	Units	Formula
m_1	Kg	$m_1 = m_t + 0.5 m_s$
m_2	Kg	$m_2 = m_c + 0.5 m_o$
d_1	metres	See comments below.
Q	-	$Q^2 = 1.08 \text{ x } 10^{-12} m_1 L_1^3 / d_1^4$
a	-	$a = L_2 / L_1$
b	-	$b = m_2 / m_1$
c	-	$c = (d_2 / d_1)^4$
A_1	-	$A_1 = \frac{a^3 b}{3c} + \frac{a^2 b}{3} + \frac{1}{48}$
A_2	-	$A_2 = \frac{b}{144}[\frac{a^3}{c} + \frac{7a^2}{16}]$
S	-	$S_{1,2} = \sqrt{\frac{[A_1 \mp (A_1^2 \quad 4A_2)]^{\frac{1}{2}}}{2A_2}}$
N_c	rev/min	$N_{c1} = S_1 / Q$ and $N_{c2} = S_2 / Q$

These formulae can be used in either of two ways:-

1. Approximate values for the critical speeds can be obtained by using a value of d_1 (the effective diameter of the mid-bearing shaft span) taken from scale drawings.

2. If the first critical of a machine without a coupling mass is known, either from manufacturer's information or from calculations using conventional analyses techniques, then this value 'N_{co}' can be used to estimate shaft diameter d_1 as follows:-

$$d_1^4 = 2.248 \text{ x } 10^{-14} N_{co}^{\ 2} . m_1 L_1^3 \quad \text{(metres) for steel shaft.}$$

This value of d_1 will therefore take into account any shaft stiffening features, such as seals, which have been included when determining N_{co}.