Forces generated by Gear Couplings

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SYNOPSIS

Gear couplings can apply forces and/or moments of considerable magnitude to the machines which they connect, and the resulting bearing loads and shaft deflections need to be considered because of their effect on the machines components and the system dynamics.

The forces of greatest practical significance are:

1. Axial forces which are transmitted through gear couplings due to tooth friction, and have not infrequently given rise to thrust bearing failures, due to inadequate allowance being made for them in design.

2. Those arising as a consequence of transverse moments generated at each mesh due to misalignment, which can be high enough to overload adjacent radial bearings as well as being a cause of bearing vibration in some conditions, but are often not allowed for at all in design.

3. Those due to unbalance, which can be seriously intensified by shaft end deflections, and by eccentric running or “spacer throw out” at the meshes under high speed, low torque operation.

In this paper the origins of the various forces are explained, factors which influence them are discussed and guidance on their estimation for system design is given.

AXIAL FORCES

When a coupling is transmitting torque, relative movement between the mating teeth is resisted by friction at the tooth contacts, and for smooth lubricated steel surfaces the coefficient of friction in slow sped sliding at any individual tooth contact will be about 0.15. However, the apparent friction coefficient for the whole coupling due to externally imposed axial displacement – for example, due to thermal expansions of the connected machines – depends on the sum of all the tooth friction forces, and when the maximum sliding velocity Vm associated with any angular misalignment present, is greater than the sliding velocity associated with axial displacement of the coupling, Va, as will normally be the case, some of the individual tooth friction forces will actually be assisting the axial displacement, as shown in Fig. 1. For this reason the net friction force may be much less than 0.15 x (net tooth force due to torque transmission) and values of 0.05 or less for the apparent friction coefficient are not uncommon.

On the other hand, if the angular misalignment, and hence Vm, is small, and the axial expansion, Va, is quite rapid, for example due to the sudden application of thrust to one of the connected shafts, all the tooth friction forces may oppose the movement, and the effective friction coefficient may then approach 0.15.

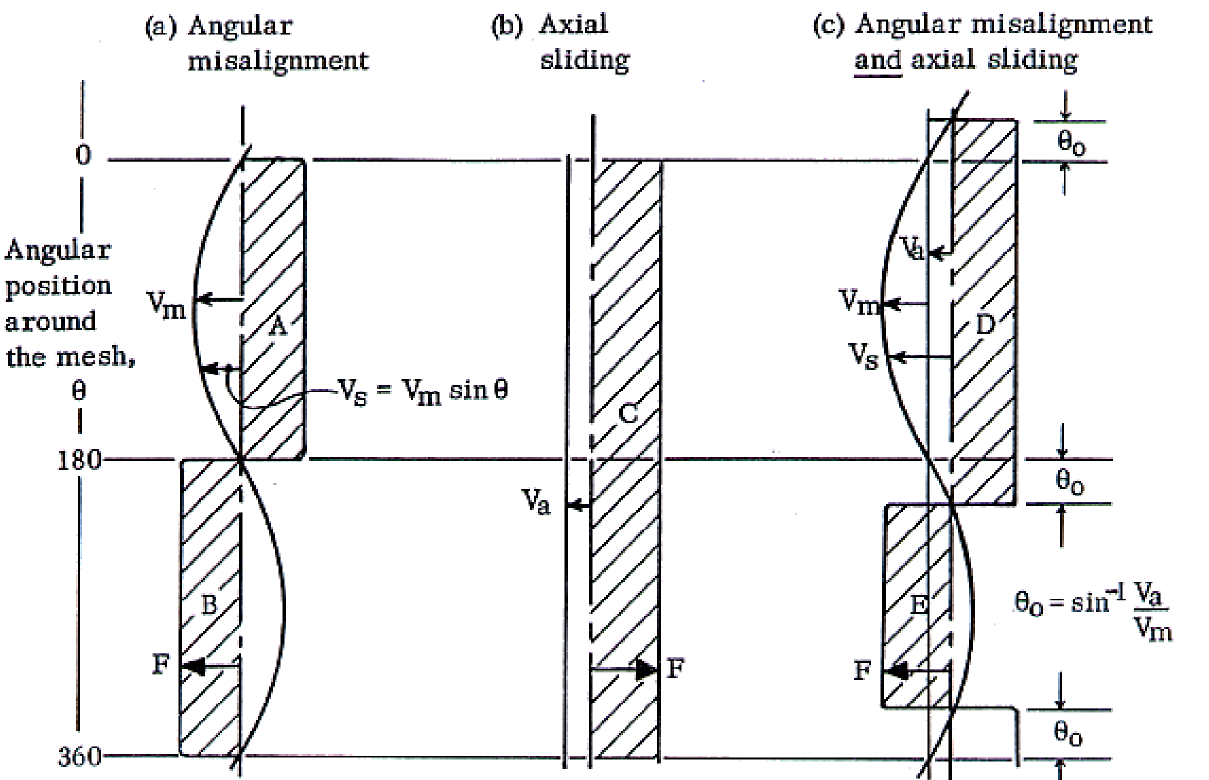
There are also various circumstances in which the effective friction coefficient may be higher than 0.15. Values up to 0.3 have been reported, and many thrust bearings designed for $\mu = 0.15$ have failed. This has led to an API specification for thrust bearings for $\mu = 0.3$. For larger high speed machines, however, this figure makes the thrust bearing design very difficult, due to excessive power loss and oil temperature rise.

Factors which can increase the friction coefficient, and which probably account for many of the observed thrust bearing failures, are a measure of interlocking of the teeth due to major damage, poor surface finish due to an inadequate standard of manufacture and inadequate lubrication of the contact areas. In practice, certainly for critical applications, it is desirable from considerations of both friction and backlash increase, to keep the total amount of wear to a minimum by using the best available materials and designs, and by changing the couplings before major wear damage begins to develop. High quality manufacture will avoid the problem of poor initial tooth finish, and continuous oil feed to each tooth, with a lubrication system designed for high reliability, would minimise the danger of inadequate lubrication.

Unfortunately even when all these points are taken care of, there still remains a possible cause of a friction coefficient developing in excess of 0.15, namely near-perfect alignment at each mesh, resulting in no relative movement between the teeth.

In this case oil could become totally excluded from the contact areas, and although wear would be negligible, the friction could increase to that for dry surfaces, around 0.3. A subsequent axial movement could then lead to thrust bearing failure. It should be noted that the angular alignment does not have to be perfect for the teeth to lock since the shafts and spacer are capable of bending slightly to cater for slight misalignment. This emphasises the need for rigid coupling designs, and also implies that one possible solution is to build in misalignment during line-out of the machinery such that the near-perfect alignment situation will be avoided. In practice

Patterns of local tooth sliding speed, V_s, and friction force, F, with :-



Net axial force is proportional to :- Area A - Area B = 0

Net axial force is proportional to Area C and equals:- $\frac{\mu T}{R}$

Net axial force is proportional to Area D - Area E and equals:-

$$\frac{\mu T}{R}\left(\frac{180+2\theta_o}{360} - \frac{180-2\theta_o}{360}\right)$$

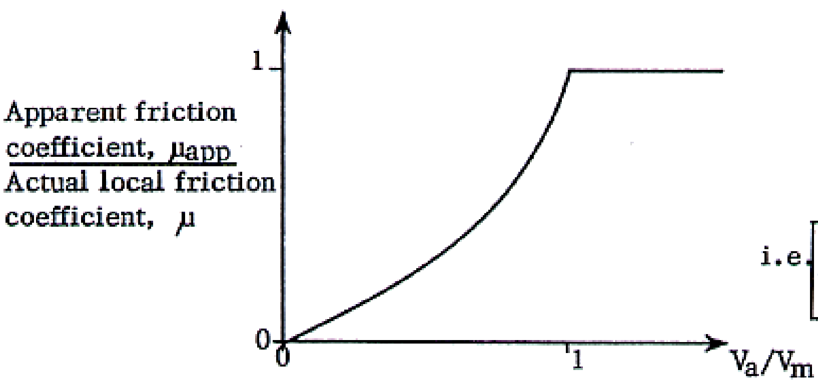
$$= \frac{\mu T}{R}\left(\frac{\theta_o}{90}\right) = \frac{\mu T}{R}\left(\frac{\sin^{-1} V_a/V_m}{90}\right)$$

For the general case, (c), the apparent friction coefficient, $\mu_{app} = \frac{\text{Net axial force}}{\text{Total tooth force}}$

$$= \frac{\frac{\mu T}{R}\left(\frac{\sin^{-1} V_a/V_m}{90}\right)}{T/R}$$

$$= \mu\left(\frac{\sin^{-1} V_a/V_m}{90}\right)$$

i.e. $\boxed{\frac{\mu_{app}}{\mu} = \frac{\sin^{-1} V_a/V_m}{90}}$



EFFECT OF RATE OF AXIAL SLIDING ON THE APPARENT FRICTION COEFFICIENT IN A GEAR COUPLING. FIG.1

this is probably almost impossible within the very tight angular misalignment limits currently specified for high speed couplings (about 1mm/m), but would be feasible if the limits could be extended by a factor of about three, as discussed in the first paper to this conference.

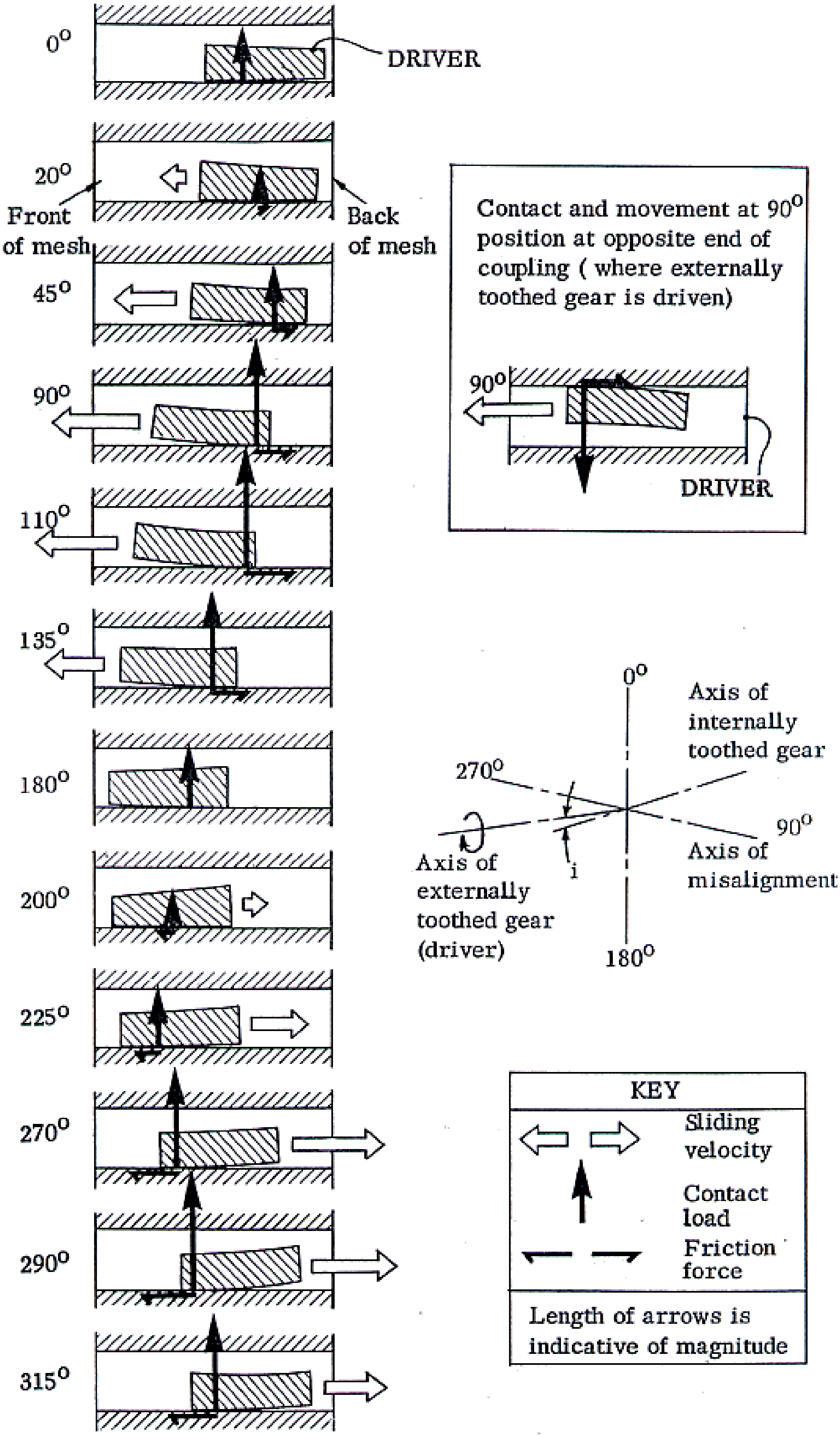
If the possibility of near-perfect alignment cannot be avoided, three options are available. Firstly, where a very high level of reliability is essential, the thrust bearing must be designed for $\mu = 0.3$. If this makes the trust bearing design marginal, it may be possible to overcome the problem with a larger PCD coupling, although the extent to which this can be dome may be limited by the increased weight, tooth sliding speed, or friction moment. This reduces the net tooth force and hence the thrust transmitted with a given friction coefficient. If, even then, an adequate thrust bearing design cannot be produced, the second option is simply to accept the possibility of very infrequent thrust bearing overload and to design for a lower friction coefficient of, say, 0.15. The difficulty here id that the risk of failure cannot really be properly quantified and if this option is unacceptable, the only remaining alternative is to use a different coupling design altogether.

Another interesting feature of gear couplings, connected with their axial force generation, is that the spacer will generally be driven to one end of its free travel by the small but real force which arises as a result of spacer weight and tooth friction. The spacer weight causes the net tooth force in the 0-180° sector to differ from that in the 180° - 360° sector and consequently, referring to Fig. 1(a), area A, representing the net friction force in the 0-180° sector, is actually slightly different from area B so that a net friction force results. In the common parallel offset situation it can readily be shown that the net friction forces on the spacer at each mesh are additive and would therefore move it to the end of its travel. This has been observed in practice by Boylan (Ref. 1).

TRANSVERSE FORCES DUE TO MISALIGNMENT

Substantial transverse forces may act on the shaft ends connected by a gear coupling when any misalignment is present, due to transverse moments which are generated at the meshes. The way these moments arise can be illustrated by reference to Fig. 2, which is taken from the first paper to this Conference (Ref.2).

This figure shows diagrammatically the condition of local tooth loading, contact and movement around the mesh of a misaligned coupling. It can be seen, firstly, that the contact loads on the male gear teeth act nearer the 'back' end of the teeth from 20° - 200°, and nearer the front end of the teeth from about 200° - 380°. They produce a moment on the male gear about the axis of misalignment, and a corresponding opposite moment will act on the female gear. This moment will be referred to hereafter as the tilting moment. Secondly, it can be seen that the friction forces on the male gear teeth act towards the back of the mesh from 0° - 180°, and towards the front of the mesh from about 180° - 360°. This will produce a friction moment on the male gear approximately about the 0° - 180° axis, and a corresponding opposite moment on the female gear. A third moment, associated with turning the torque through the misalignment angle, also exists, but is equal to torque x misalignment angle (in rads) and is consequently negligible in high speed, high power applications since misalignment angles have to be restricted to such small values (less than about .002 rads).



CONDITIONS OF TOOTH LOADING, CONTACT AND MOVEMENT AT A MISALIGNED MESH OF A GEAR COUPLING FIG.2

Only the first two moments need therefore be considered in this paper. Both are directly proportional to the torque, and both depend somewhat on the uniformity of load distribution and the amount of misalignment and crowning.

Maximum moments occur when the coupling is appreciably misaligned so that the torque is transmitted by the teeth in two zones on the (90 + Ø)° / (270 + Ø)° axis (where Ø is the pressure angle). This situation is illustrated in Fig. 3, which also gives the associated simple expressions for the tilting and friction moments. Note that in this case the axis about which the friction moment acts is displaced by Ø° from the plane of misalignment.

When the misalignment is lower and the load distribution is consequently more uniform, the moment arms of the individual tooth forces are reduced. Also, the contact points tend to lie closer to the centre plane of the gear, and this effect is enhanced by the use of crowning. The results of an approximate analysis of these effects is summarised in Fig. 4, which shows the tilting and friction moments (as ratios of the transmitted torque), for various degrees of misalignment, and for straight and for correctly crowned teeth.

When the misalignment is small the axis about which the friction moment acts will be closer to the plane of misalignment than that shown for large misalignment in Fig.3. For most practical purposes it appears reasonable to consider that the friction moment acts about an axis perpendicular to the axis of misalignment. The two moments therefore act approximately at right angles, and have a resultant moment which can be obtained by vectorial addition, as indicated in the top right sector of Fig. 4.

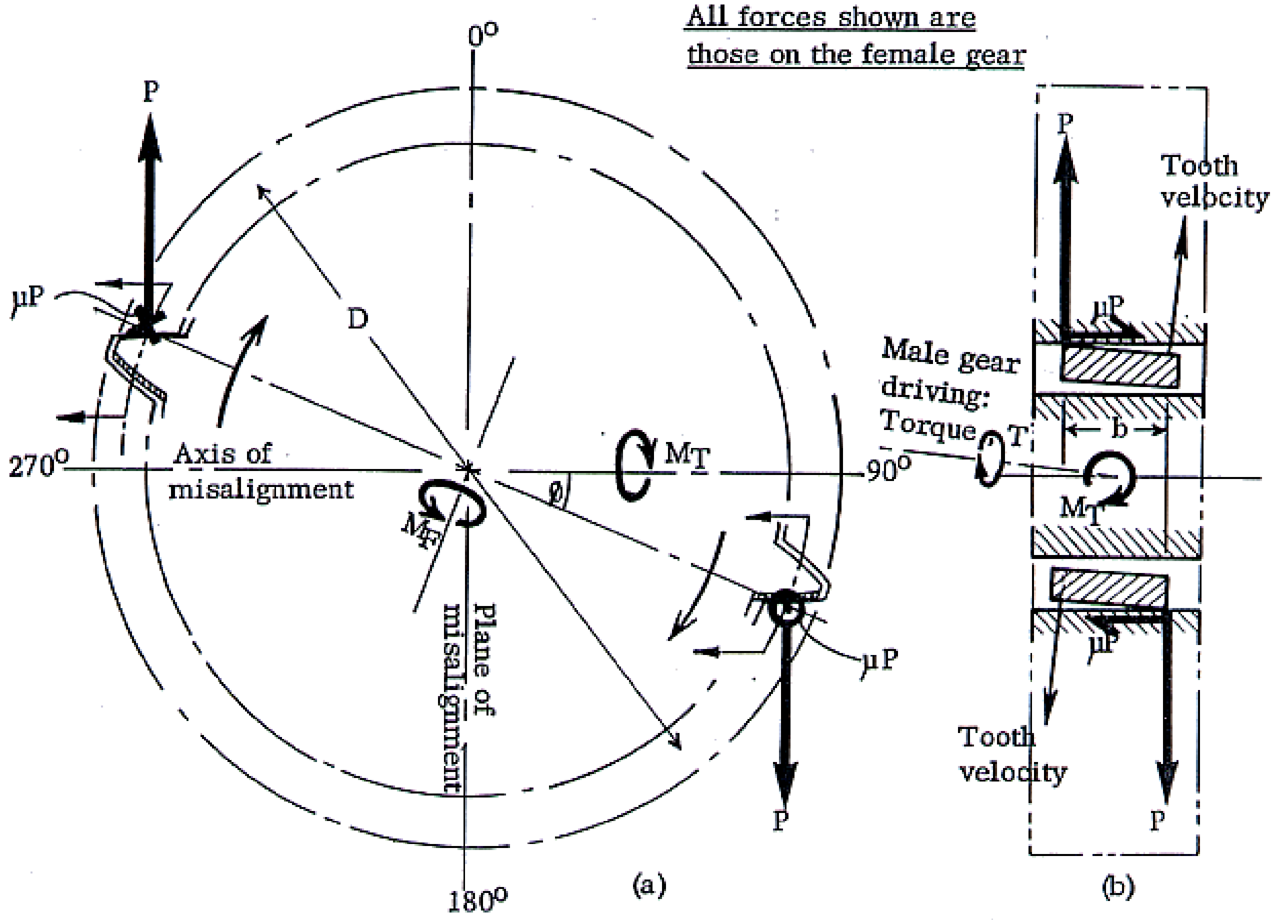
For low misalignment couplings, typical values for the resultant moment, based on Fig. 4 and maximum likely values for b/D and μ of 0.2 and 0.12 respectively, are:

$$M_R/T = 0.12, \text{ for crowned tooth couplings.}$$

$$M_R/T = 0.16, \text{ for straight tooth couplings.}$$

and these values are suggested to be appropriate for initial design guidance. The resulting actual values of M_R are shown as a function of torque in the bottom left corner of Fig. 4, and their appreciable magnitude may be noted. Similar values have been found experimentally (Refs. 3, 4).

The main significance of the transverse moments due to the misalignment at each mesh, is their effect on the forces on the adjacent bearings supporting the connected shafts. The moment at each mesh has two effects. Firstly it acts directly on the shaft, causing one set of bearing reaction forces to appear. Secondly, it acts on the spacer and calls for transverse reaction forces at each mesh to maintain equilibrium of the spacer. The associated force acting on the shaft end at each mesh then produces a second additive set of reaction forces at the machine bearings. The situation is illustrated for a 2-rotor, 4-bearing system in Fig. 5. For clarity the effect of a misalignment moment at the left hand mesh only is shown.



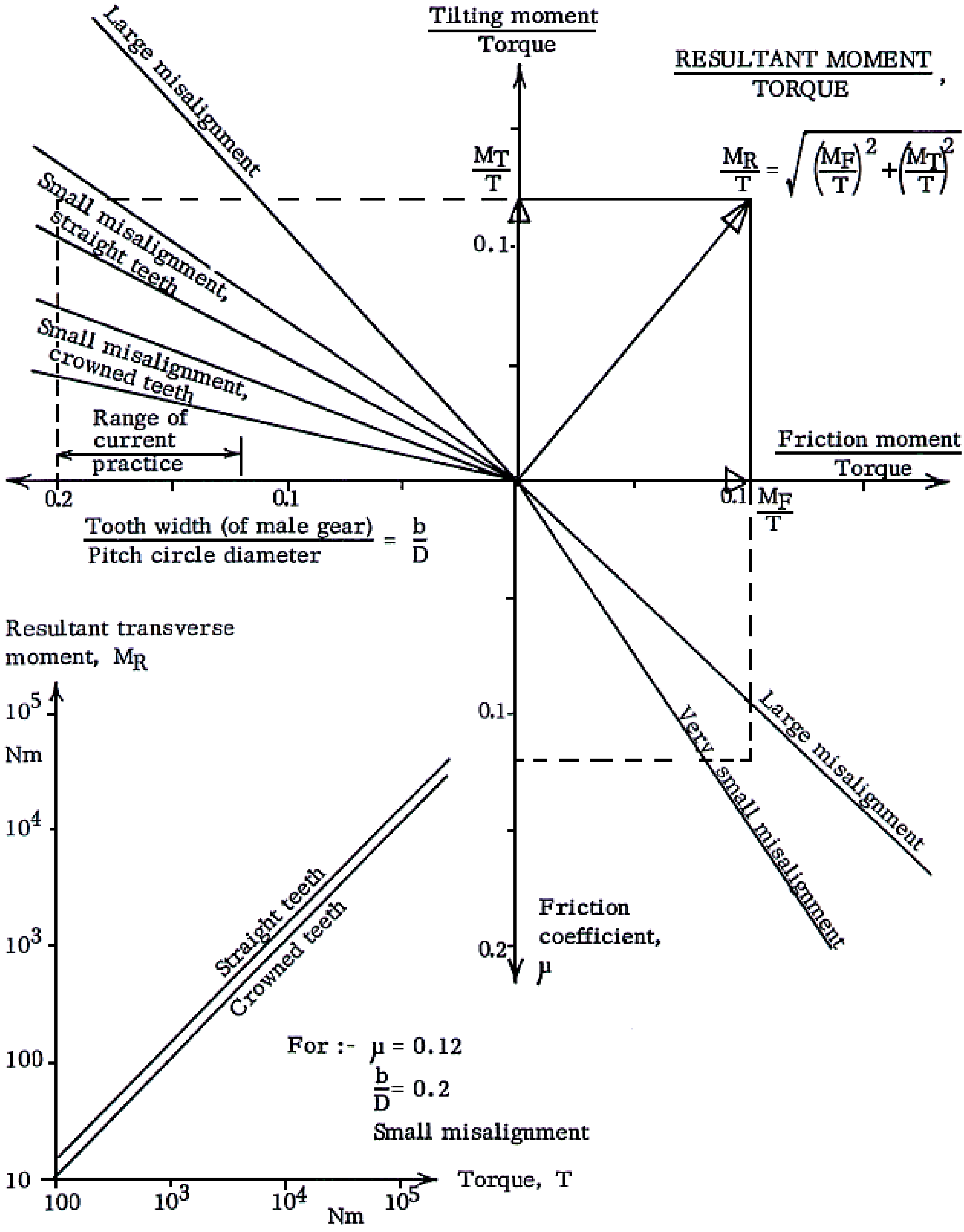
Tooth force, $P = \dfrac{\text{Torque, } T}{D \cos \phi}$ (from (a))

Tooth friction force $= \mu P$

Tilting moment, $M_T = P \times b = \dfrac{b}{D \cos \phi} \times T$ (from (b))

Friction moment, $M_F = (\mu P) \times D = \dfrac{\mu}{\cos \phi} \times T$ (from (a))

TILTING AND FRICTION MOMENTS ON THE FEMALE GEAR OF AN APPRECIABLY MISALIGNED GEAR COUPLING FIG. 3



TRANSVERSE MOMENT GENERATED BY A MISALIGNED GEAR COUPLING

FIG. 4

In general, however, a second moment would act at the right hand mesh, the magnitude of which would be about the same, but the direction of which would depend on the direction of the angular misalignment at the second mesh relative to that at the first mesh. In the worst case the two moments would have a directly additive effect. This, in fact, occurs where the shaft ends are parallel but offset, a situation which is frequently approached in practice.

Consequently, referring to Fig. 5, the load due to coupling misalignment on the adjacent bearing of the first machine can have a maximum value of:

$$R_1 = P\left(\underbrace{1 + \frac{L + a_1}{b_1}}_{\text{Force due to moment at adjacent mesh.}}\right) + P\left(\underbrace{1 = a_1/b_1}_{\text{Force due to moment at other mesh.}}\right) = \frac{M_R}{L}\left(2 + \frac{L + 2a_1}{b_1}\right)$$

where a_1, b_1, and L are as shown in Fig. 5, $(L + 2a_1)/b_1$ would not generally be much more than ⅓ so that, using this value,

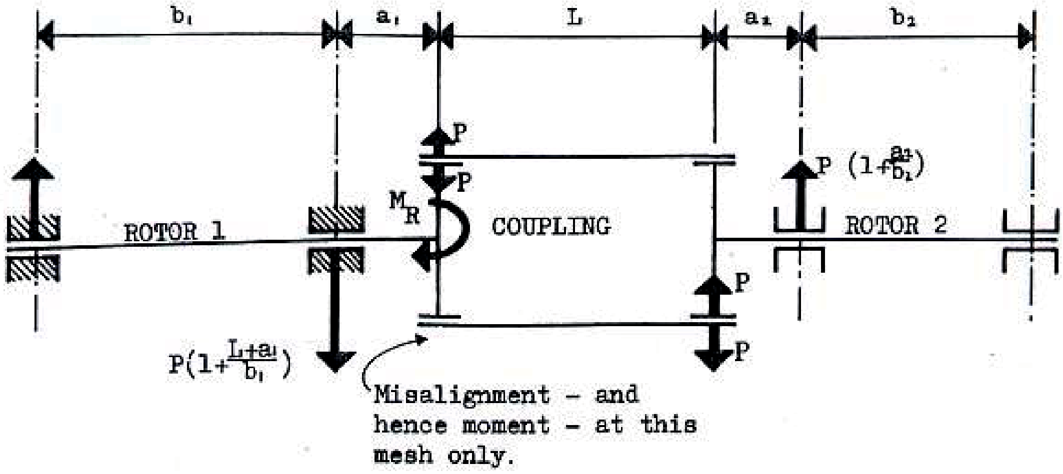
$$R_1 = 2\frac{1}{3} \times \frac{M_R}{L}$$

Using the values for M_R suggested in Fig. 4, the resultant bearing loads are as shown in Fig. 6. For example, for a coupling of 0.6m length transmitting a torque of 25,000 Nm the bearing load would be about 12,000 N for crowned tooth couplings, and 16,000 N for straight tooth couplings, at the maximum design misalignment. For shorter couplings the loads would be correspondingly increased. The direction of the resultant bearing load, in this case of parallel offset shaft ends, lags the direction of relative oddest of the other machine by $\tan^{-1}$ (M_F/M_T)m typically about 40°m as shown in Fig. 6.

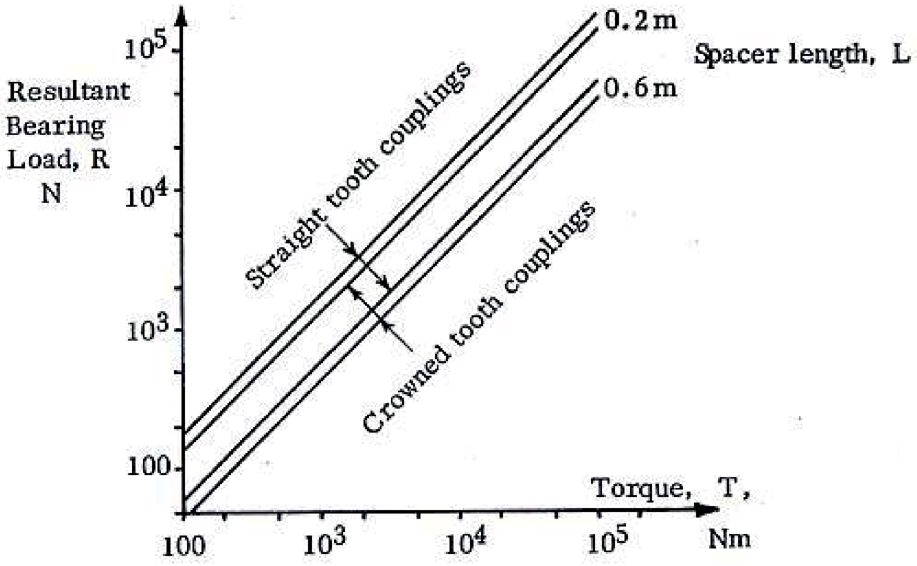
The significance of these forces is primarily for their possible effect in the adjacent machine bearings, since they could well be of the same order as, or even greater than the static bearing loads. If acting upwards, the forces could sufficiently unload the bearings to promote bearing instability. Although such a phenomenon does not appear to be recognized in practice, it could account for some of the vibration problems experienced with gear couplings in service.

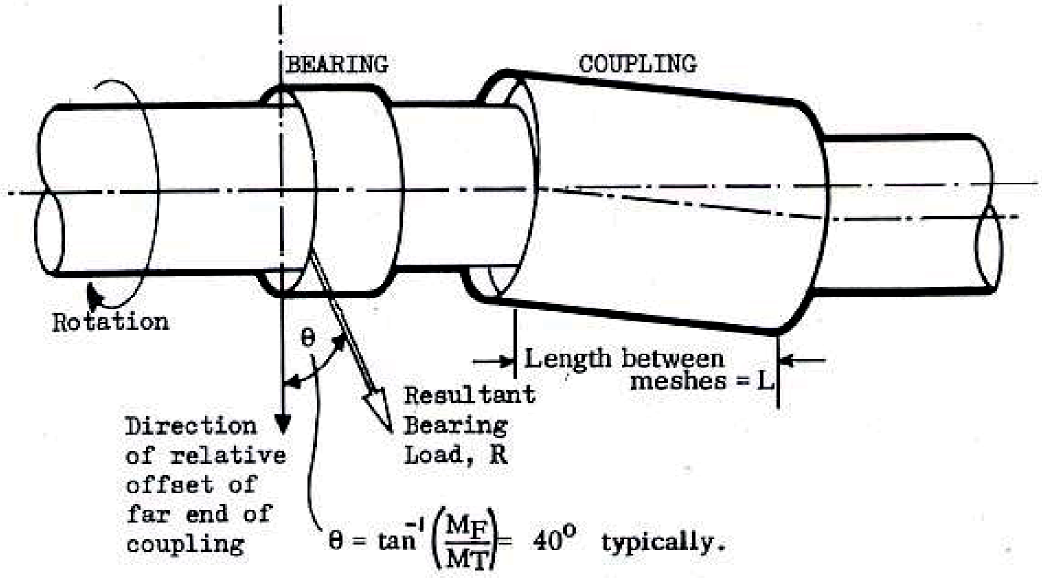
The transverse forces have also to be transmitted by the coupling gear meshes bit would only increase tooth loading by up to about 10%.

In connection with the friction moment, an interesting discovery reported in Ref. 3, was that at low misalignment the coupling meshes could lock and, instead of sliding, could require the shaft ends to bend to accommodate the misalignment. This then resulted in a rotating moment being applied to the shaft ends, and a rotating force on the bearings. This provides another possible cause of unaccountable vibration problems in systems with well balanced parts, and also increases the range of misalignments capable of giving friction lock, which is important in relation to axial thrust transmission as discussed earlier. This effect will be alleviated by the use of a stiff spacer and shaft ends, and reversed hubs.



FORCES IN A TWO-ROTOR, FOUR-BEARING SYSTEM DUE TO A MISALIGNMENT MOMENT AT ONE MESH OF THE COUPLING FIG.5





MAGNITUDE AND DIRECTION OF THE RESULTANT BEARING LOAD FOR A PARALLEL MISALIGNED COUPLING FIG. 6

Additional transverse forces can also arise due to coupling unbalance and these are considered further in the next section.

TRANSVERSE FORCES AND LATERAL VIBRATIONS – DUE TO UNBALANCE

Gear couplings are prone to lateral vibration problems, and good design and accurate manufacture are required if they are to be avoided. One of the main difficulties is that of achieving good balance because the coupling cannot generally be balanced in the assembled condition, since the spacer is a loose part and must transmit torque if it is to be properly centred.

The best that can normally be achieved is a balancing of the hubs, and of the spacer separately, using its OD as a reference diameter on the balancing machine. The balance of the whole assembly in the torque-centred running condition then depends on the degree of concentricity between the various gear pitch circles and the balancing reference diameters, and the degree of freedom from cumulative pitch errors. Both must be very high for really good balance. There would seem to be a good case for developing techniques for balancing the complete coupling, for example by using a torsion bar to lock torque into the coupling.

As with other couplings, when a three piece spacer is used, the location between the parts of the spacer must be accurate and repeatable, using either spigots or fitted bolts. Fitted bolts may be preferable for high speed applications because of possible loss of spigot interference due to relative centrifugal growth of the parts.

The fact that the spacer will be subject to some degree of eccentricity also emphasises the need for low spacer mass. It is therefore important to use the smallest coupling size for the required duty, and the spacer must be in the form of a tube if high strength material with the lowest wall thickness consistent with reasonable resistance to handling damage and acceptable shear stress. A wall thickness of about 5mm is probably about the minimum acceptable for use in an industrial application. For some machines, considerable weight saving can be achieved by using a single piece spacer, but this greatly complicates assembly and disassembly, and is often impracticable.

Low spacer mass, together with low overhung length, is also important to minimise the reduction of the lateral critical speed of the shaft overhangs. A reversed hub design to reduce overhung length is therefore recommended, and hub mass should be minimised by use of high strength materials. Hubs fitted by oil injection will also be advantageous since avoidance of keys should allow smaller hub OD's to be used.

A significant merit of gear couplings is that an oil seal between the bearing and the coupling is not necessary, provided both use the same oil system, and advantage can be taken if this feature to further reduce overhung length. With attention to these various aspects, gear couplings provide the lowest overhung moment of any coupling type, and this will be an important factor when choosing couplings for high speed applications.

An empirical limit has been established for the spacer mass and PCD of gear couplings, based on an extensive survey of gear coupling performance mainly in marine service, by Conti-Barbaran (Ref.5). This limit is defined in terms of a parameter (now commonly referred to as the Conti-Barbaran parameter), which is:

$$\varepsilon = \frac{HP \times 10^{15}}{\text{Spacer mass (kg)} \times (\text{rpm})^3 \times (\text{PCD(mm)})^2}$$

Conti-Barbaran found that for values of ε below about 4, coupling performance was unsatisfactory because of rapid tooth damage, and he recommended that ε should desirably be kept above 10.

No satisfactory physical explanation of the limiting Conti-Barbaran parameter has been put forward. The parameter is closely related to the ration of the net tooth force due to torque and the spacer centrifugal force acting at each mesh.
In fact:

$$\frac{F_{T_O}}{F_{C_O}} = \frac{.0026}{a/D} \times \text{Conti-Barbaran parameter.}$$

where a is the whirl radius of the spacer mass centre,

and F_{T_O} = net tooth force to torque = full running torque/pitch circle radius,

F_{C_O} = centrifugal force at each mesh = half spacer mass x (angular velocity at full running speed, rad/s)^2 x a

The whirl radius of the spacer mass centre is the spacer mass centre error associated with unbalance and concentricity errors, plus any displacement of the hub gears from the axis of rotation, due to shaft deflection resulting from dynamic effects at speed.

For a high precision coupling with a PCD of, say, 250 mm, centred by torque, 'a' might typically have a maximum value of 0.05 mm so that from the equation above:

$$\varepsilon = 0.077 \times F_{T_O} / F_{C_O}$$

Thus, in this case, when ε has its limiting value of 4, i.e. when a relatively large and heavy coupling is used, $F_{T_O} / F_{C_O} \approx 1/50$.

It is hard to see why a centrifugal force of this low magnitude, compared with the torque force, should have any significant direct effect on tooth loading and wear. One possibility is that the whirl radius might be higher than that assumed above, due to the spacer throwing out its extreme eccentric position as limited by the clearances at the tooth meshes.

This could occur due to a transient reduction in torque for example during load change. Even then however, unless the coupling teeth were already heavily worn, it is unlikely that the backlash in a high speed coupling would exceed 0.2 mm say, which means that the centrifugal force could still only be a small fraction (of the order of one tenth) of the torque force. This still seems too small to have a significant direct effect on tooth wear, and in fact many couplings which are running satisfactorily will be experiencing transverse loads of this order, due to the moments generated at the meshes.

In any case the fact that, even with the spacer thrown out, $F_C < F_T$, means that the spacer must be recentred by torque on resumption of normal running so that the 'thrown-out' condition can only occur transiently.

There seem to be two possible conclusions:

1. For some unexplained reason, the rotating load only has to exceed a very small fraction of the torque force to cause accelerated tooth wear.

2. With the Conti-Barbaran parameter at or below its critical value, the connected shaft system is on the flank of a lateral resonance which causes large dynamic deflection (whirling) of the shaft ends, and hence the spacer, which in turn causes accelerated tooth wear due to excessive angular misalignment at the meshes and/or high centrifugal loads.

In either case there would seem to be some doubt as to the general applicability of the criterion and further study appears to be needed to give it a sounder basis. In the meantime, in the absence of any more satisfactory criterion, it would appear advisable to adhere to Conti-Barbaran's recommendation of 10 as the minimum acceptable value of the parameter for use in design.

As mentioned above, coupling spacer 'throw-out' can occur if the transmitted torque falls to a low value in relation to the speed. In many drives including most pump, compressor and propellor drives, this can only occur transiently since the torque transients are not a regular feature of operation, satisfactory running should be obtained without recourse to tip centring. If tip centring is used, the tooth tips need to be accurately produced if interference between the tip centring and flank centring is to be avoided, and subtle design may be necessary to maintain effective tip centring at high speed because of the tendency for the female gear to grow relative to the male gear due to centrifugal strains. For these reasons, tip centring is probably best restricted to applications where there is a definite low-torque/high-speed running requirement, and to provide a quantitative guide it is suggested that tip centring need only be considered where:

$$\frac{T/D}{M\omega^2\delta} \leq 1 \text{ , as a non-transient running condition.}$$

where
T = torque
D = PCD
M = spacer mass
ω = angular velocity in rad/s
δ = backlash at gear meshes.

TORSIONAL VIBRATIONS

The torsional stiffness per unit length of gear couplings is similar to that of the connected shafts, and torsional critical speeds must therefore be assessed by treating the coupled rotors as a distributed mass system.

It will rarely be possible to appreciably increase the critical speed by adjustment of the coupling dimensions alone, and any adjustments – except reducing the spacer length – will normally be at the expense of increased coupling mass, and hence reduced lateral critical speed. Although large diameter spacer tubes are theoretically able to give a better compromise between weight and stiffness, this is only obtainable if the wall thickness is simultaneously reduced, and this may not be possible to any great extent without adversely affecting the tube robustness and resistance to handling damage. For calculating the torsional vibration of the system, it will be necessary to know the torsional stiffness values of any couplings considered, and these are in fact always obtainable from manufacturer's literature.

It has been pointed out by Wright (Ref.6) that angularly misaligned gear couplings can generate a cyclic torque fluctuation at twice running speed due to cumulative pitch errors in the two gears of a mesh. This suggests the desirability of keeping the torsional critical speeds of the system away from twice the running speed. In practice, for precision couplings for high speed drives, it seems unlikely that the degree of excitation due to this cause will be large.

CONCLUSION

By way of conclusion, the various forces and related parameters for a coupling for a typical high speed, high power application are given below:

Coupling: Power = 17 MW = 22,800 HP
Speed = 6,500 rpm
Torque = 25,000 N

With 20° pressure angle, and case hardened teeth of width equal to 0.2 x PCD:

PCD = 0.227 m (min)
Length = 0.5 m
Spacer mass = 40 kg (light weight design)

Performance:

Transverse force on adjacent machine bearings
16,000 N with straight teeth and
12,000 N with crowned teeth

Axial thrust = 33,000 N with $\mu = 0.15$, up to 66,000 N with $\mu = 0.3$

Conti-Barbaran parameter = 40

Torsional stiffness = 7×10^6 Nm/rad

The transverse and thrust loads are clearly substantial. The transverse loads would not present any particular problem, provided they were allowed fro (as a multi-directional load) in the bearing design, and this probably applies for the large majority of applications. On the other hand the very large and potentially bi-directional maximum thrust load associated with $\mu = 0.3$ is near the limit of thrust bearing capacity at this speed and would necessitate a really well engineered high-performance thrust bearing design, which would be correspondingly expensive. This conclusion is also probably fairly widely applicable to high speed, high power plant, and is an important factor to consider in the selection of coupling type.

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