INTRODUCTION

Gear couplings cater for misalignment by means of mating external and internal spur type gears, having a small amount of clearance. Axial sliding is unrestricted except by friction, whilst angular misalignment is permitted by tilting between the teeth and is limited, ultimately by take up of the tooth clearances, and more or less earlier by local overloading or excessive sliding speeds at the tooth contacts.

A variety of different forms are used, depending on assembly requirements, distance between shaft ends, method of lubrication, etc., and a selection of general types is shown in Fig.1. The couplings are all shown with three piece spacers, but two or one piece spacers are also used where assembly and disassembly will allow.

Involute tooth profiles are used almost exclusively, principally because they can be produced on standard gear manufacturing equipment. For applications requiring large angular misalignment, they also have the advantage, when combined with a suitable crowning method, of providing uniform velocity transmission. Pressure angles of 20° - 25°, and 75% to 70% full depth teeth, are generally used for high torque, high speed couplings.

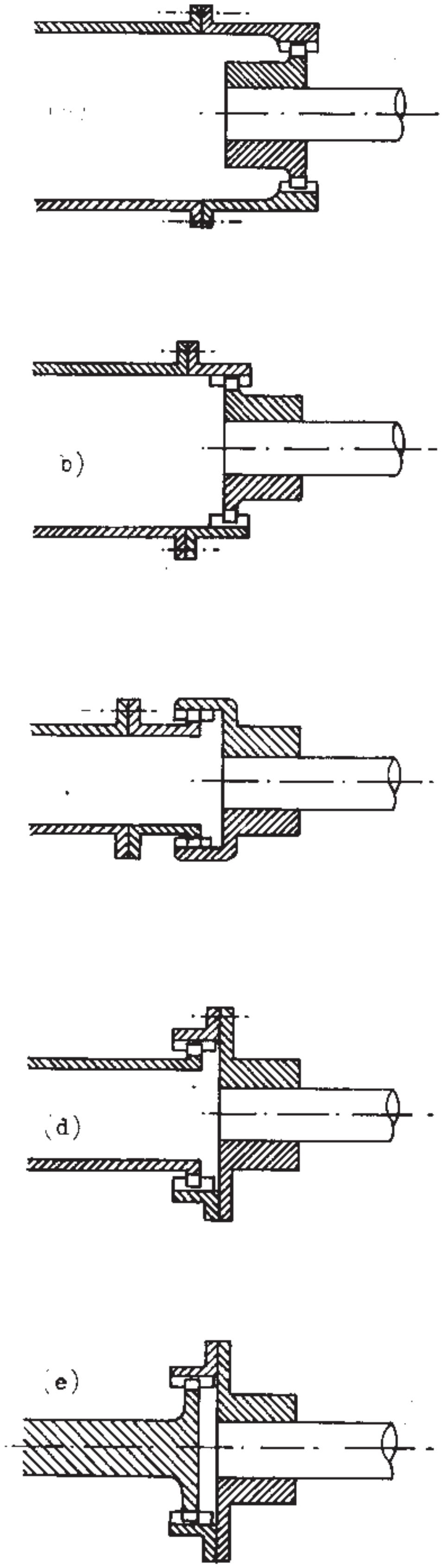
Various methods are employed for crowning the axially shorter teeth of a mesh (almost always the male teeth) to increase misalignment capacity. The method required for crowning involute teeth to provide uniform velocity transmission at large angles (up to about 10°) is described in detail by Renzo (Ref.2). For large, high speed couplings, however, allowable angular misalignments are restricted to a few minutes by a limit on tooth sliding speed, and for these application, crowning, if used at all, need only be very slight and can be produced in a less specific manner, since any resulting fluctuations in transmitted velocity will be quite insignificant.

GEOMETRIC ACTION AND THE NATURE OF CONTACT AT THE TEETH

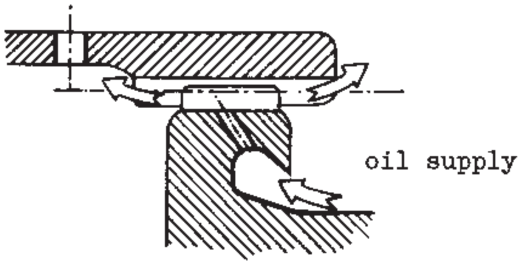
Before considering the quantitative design of gear couplings it is useful to gain some understanding of their geometric action, and the resulting loading and movements at the tooth contacts.

With involute tooth profiles, the two gears of a mesh have teeth which can be viewed as having been generated from a common base circle. When the two gears are in aligned concentric contact, the radii of curvature of the contacting tooth surfaces are identical and, with uncrowned tooth gears, the teeth make full area contact. When the externally toothed gear, hereafter referred to as the male gear, has crowned teeth, the contact is theoretically along a line only, but in practice, under load, contact spreads over a more or less wide zone, depending on the load and degree of tooth crowning (see Fig.2).

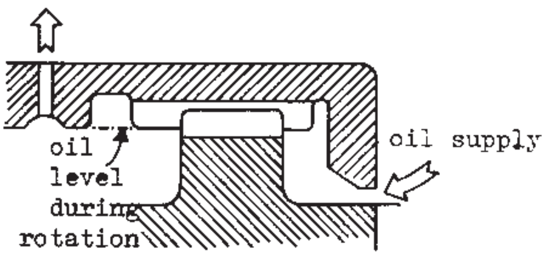
Normally, of course, the meshing gears will be more or less misaligned and, referring to Fig.3, it can be seen that when misalignment is imposed, the mating tooth flanks only tilt relative to one another in two positions around the mesh, whereas in the two positions on the perpendicular axis the teeth only slide relative to one another. At intermediate positions there will be a combination of tilting and sliding.



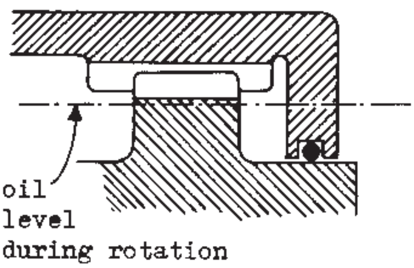
Basic Configurations
- using 3 piece spacers



Continuously Lubricated
- Damless Design



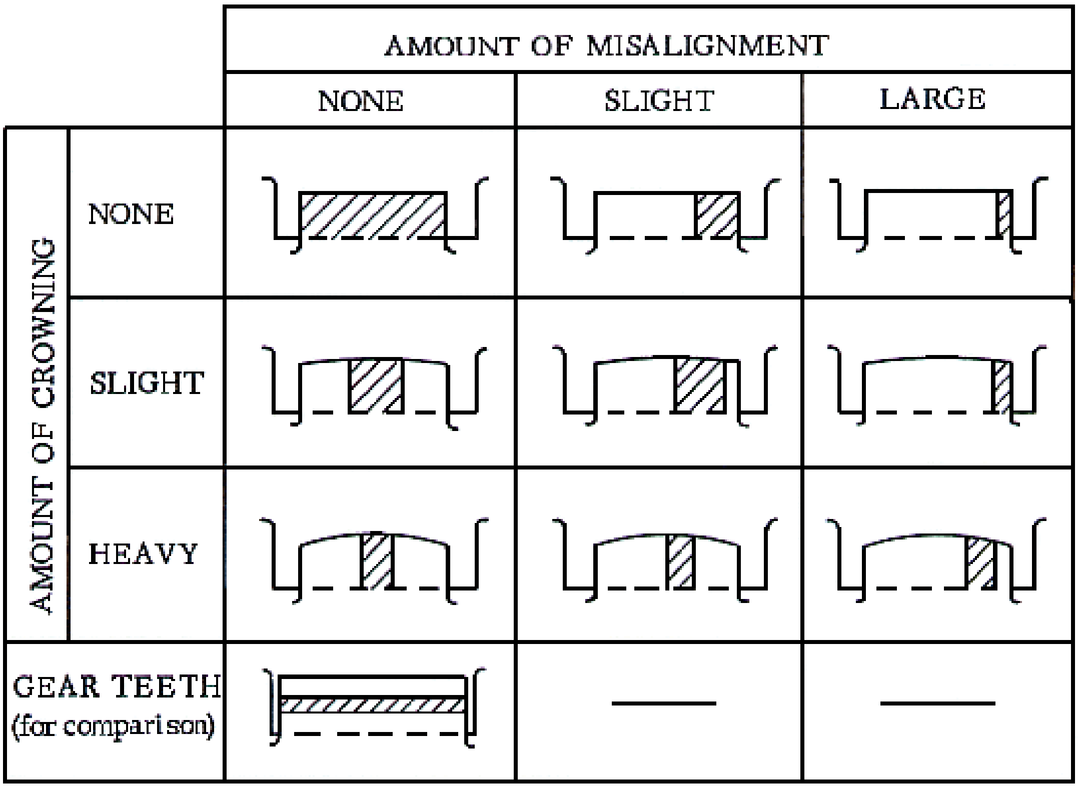
Continuously Lubricated
- with Lubricant Dam



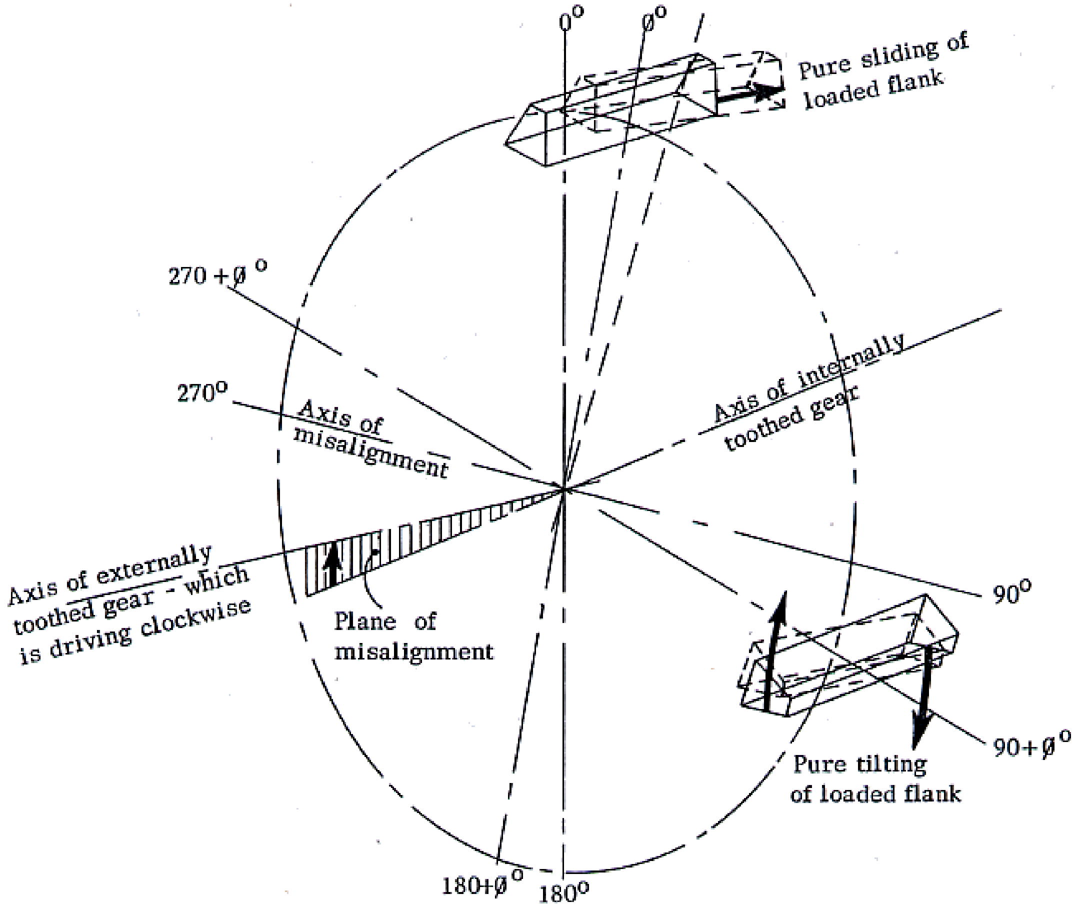
Grease or Oil Filled

Methods of Tooth Lubrication

GEAR COUPLING DESIGN FORMS FIG.1



EFFECT ON TOOTH CONTACT AREA OF THE AMOUNTS OF MISALIGNMENT AND TOOTH CROWNING
FIG. 2



TOOTH MOVEMENTS DUE TO MISALIGNING A GEAR COUPLING MESH FIG.3

The tilting action brings the mating flanks into closer contact at one end, and causes the affected tooth pair to carry a greater share of the coupling load. Greatest relative tilt occurs at the circumferential position where the loaded flank is perpendicular to the plane of misalignment, that is, at angular positions of 90 + Ø (see Fig.3), where Ø is the tooth pressure angle. The teeth at these positions carry the highest tooth loads. (Note that the angles are measured from the plane of misalignment in a circumferential direction which is defined by going from the centre of the male tooth through whichever flank is loaded by the torque).

Half way between the positions of maximum tooth load, that is at the 180 + Ø° positions, the planes of the loaded flanks are parallel to the plane of misalignment so that the flanks remain parallel during misalignment, and only slide relative to one another. The teeth at these positions carry least load. Typical patterns of tooth load distribution are shown in Fig.4. As shown, the ratio of the peak to average load can be considerably greater than one, and because peak load acts when the teeth are at the positions of maximum relative tilt, the ratio of peak to average tooth stress will be even greater. Typical patterns of tooth contact for different degrees of crowning and misalignment are shown in Fig.2 for one of the two positions of maximum tooth load. At the second position of maximum tooth load the contact areas would be positioned on the opposite side of the tooth mid point at an equal distance from it.

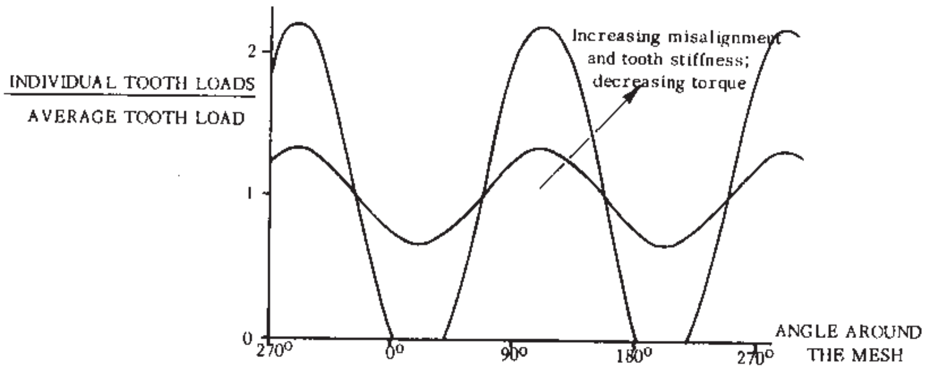
Considering now the relative movements between the teeth, it can be seen that any pair of mating teeth on a misaligned coupling, have to execute an axial reciprocating relative to one another, once every revolution of the coupling. For a typical large misaligned high speed coupling, the total sliding distance per revolution might be about 0.5 mm, and the sliding velocity might vary from a maximum, on the axis of misalignment, of 0.1m/s to zero on the perpendicular axis.

A typical complete cycle of tooth loading, contact, and movement for a 20º pressure angle straight tooth coupling is illustrated diagrammatically in Fig.5. The figure shows successive views of one tooth of the male gear, together with the mating tooth space of the female gear, as sectioned by the pitch cylinder of the female gear. The length of each arrow is indicative of magnitude.

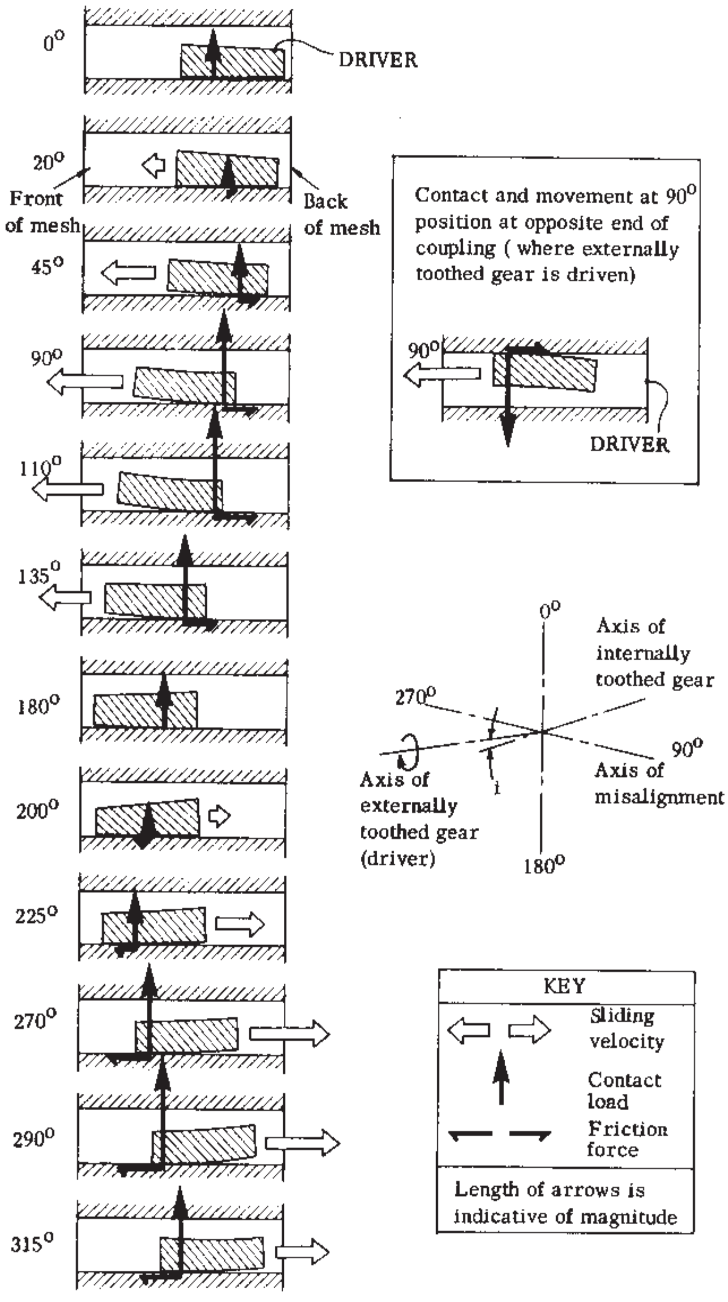
The following points may be noted:

1. Maximum load, maximum tooth misalignment, and maximum sliding velocity all occur at about the same positions and twice per cycle.

2. In the case illustrated of the short male teeth driving, the higher load regions are accompanied by highest sliding velocities and a favourable inclination of the teeth for generating hydrodynamic oil pressure, and this could alleviate the contact conditions to some extent. With the short teeth driven, however, e.g. at the other end of a symmetrical coupling, it can be seen from the inset in Fig.5 that the higher load, higher sliding speed regions, are now accompanied by an adverse tilt of the teeth and it seems likely that, with straight teeth, wear will be higher at this end of the spacer, at least initially.



PATTERNS OF TOOTH LOADING AROUND THE MESH OF A MISALIGNED COUPLING FIG.4



CONDITIONS OF TOOTH LOADING, CONTACT AND MOVEMENT AT A MISALIGNED MESH OF A GEAR COUPLING

FIG. 5

Whether this is a significant effect in practice requires further study. If it is, crowning would be beneficial, and there might be some advantage in making the spacer hermaphrodite, i.e. one end male, and the other female.

3. At each end of a tooth there is heavy contact in conjunction with sliding in one direction only. In the other direction of movement the load is relieved. This differs from the classic fretting condition, and would ideally be simulated in any testing of materials for use in gear couplings.

The figure also illustrates important points relating to the transverse moments generated by gear couplings but these will be considered in a separate paper, (Ref.3). It will be apparent from this qualititative description of coupling action that a proper analysis to establish cycles of bending and contact stress, and of relative movements, is likely to be somewhat complex. Probably the best attempt to properly quantify the situation to date is the analysis of Renzo (Ref.2) who obtains expressions for tooth loading around the cycle as a function of torque, misalignment, and tooth geometry and stiffness. The expressions relate to crowned couplings, and do not lead to a very simple design procedure, however, and the analysis does not propose any really satisfactory method of deducing critical component stresses, nor does it offer any data on appropriate allowable material stresses or tooth stiffnesses.

A fairly recent complementary analysis by Airapetov and other Russian workers (Ref.4) gives guidance on the calculation of tooth bending and contact stresses for tilted teeth, and suggests the use of limiting values based on conventional gear practice. However, this analysis again relates to crowned tooth couplings, and also assumes uniform tooth loading, and it would need to be combined with an analysis of tooth load distribution such as that of Renzo, in order to produce a comprehensive and soundly based assessment procedure. Even then it would only be applicable to crowned teeth,

In practice, simpler and more useable empirical design procedures are generally adopted for sizing and assessing couplings. The most widely used criteria are:

1. Dudley’s method and material data for assessing critical coupling stresses – which includes empirical factors for misalignment. (Ref.5)

2. The Boylan limit on tooth sliding speed. (Ref.6)

3. The Conti-Barbaran criterion for checking that coupling spacer mass is not excessive in relation to vibration problems. (Ref.7)

Although these criteria do not fully reflect the fundamental actions involved they generally give designs which perform satisfactorily in service. The next section of the paper considers the sizing and assessment of gear couplings with reference to the first two of these criteria. The implications of the Conti-Barbaran parameter are discussed in a second paper. (Ref.3)

DESIGN LIMITS AND THE EFFECT OF COUPLING PERFORMANCE

The performance of gear couplings is assumed to be limited by three principal failure conditions, namely:

> Bursting of the female gear (or sleeve), due to an excessive combined stress at the tooth roots.
>
> Wear of the tooth flanks by fretting, due to excessive tooth contact pressure.
>
> Wear of the tooth flanks by fretting or scuffing, due to excessive tooth sliding velocity.

Tooth shear is also a possible failure mode but is not, in practice, a critical condition.

Sleeve Bursting Limit

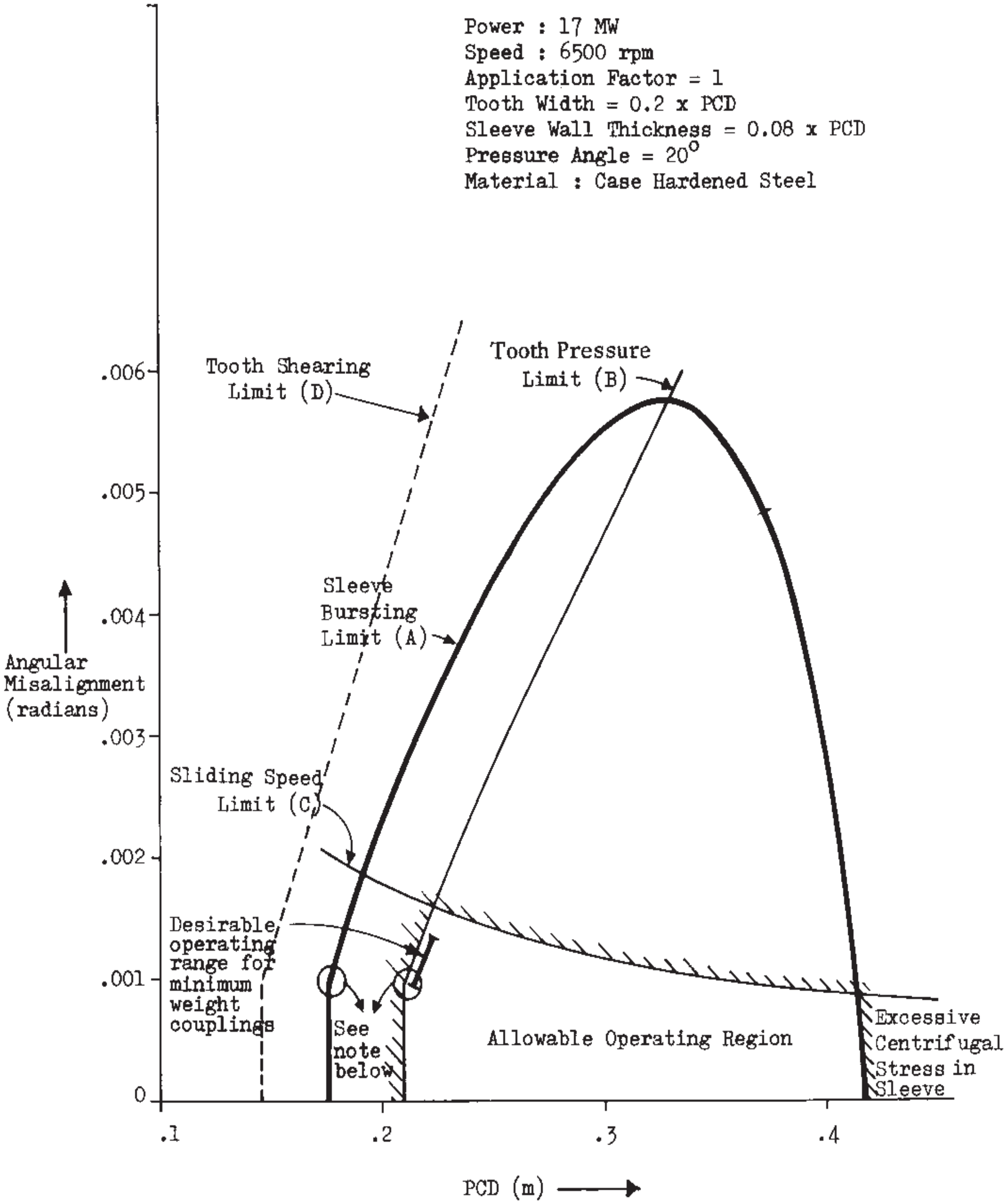
At the roots of the sleeve teeth five main stresses combine, namely:

1. Centrifugal stress.
2. Hoop stress due to the radial components of the tooth loads.
3. Tensile stress due to tooth bending under the tangential tooth loads.
4. Shear stress due to the tangential tooth loads.
5. Shear stress due to torque in the sleeve.

Dudley neglects the relatively small shear stresses, and gives an expression for the combined stress which allows for the effect of misalignment by means of two empirical factors. This expression, with a slight modification to give a better allowance for the effect of centrifugal stress, is given in the appendix and relates the maximum stress to the torque, speed, misalignment, and diameter and other dimensions of the coupling. By using typical values or functions of the diameter for these dimensions, and by using Dudley's data for the maximum allowable repeated stress values, the maximum allowable angular misalignment can be calculated and plotted, for any given torque and speed, against the coupling PCD. A typical curve is shown in Fig.6 by curve A, and defines the region within which the coupling must operate for acceptable values of the tooth root combined stress. The lower limit of PCD is determined by torque induced stress, whilst the upper limit is determined largely by centrifugal stress. The minimum allowable diameter is also limited by contact stress, however, as discussed in the next section.

Tooth Pressure Limits

The torque and misalignment on a gear coupling are limited by fretting damage and wear of the tooth flanks, which are assumed to occur when the contact pressure exceeds some critical value for the material. Dudley gives an expression for the contact pressure, on uncrowned teeth, based on the nominal stress on the overlapping tooth area factored for misalignment, and suggests maximum allowable values for a few different materials, based on practical experience (see Appendix),



Note: The 'kink' in these curves has no physical significance, - it is simply the threshold of angular misalignment below which Dudley (Ref 5) considers that misalignment has an insignificant effect.

OPERATING LIMITS FOR GEAR COUPLINGS FIG. 6

Using this expression and an assumption of a typical tooth width to diameter ratio, the minimum PCD of a mesh can be plotted as a function of angular misalignment for any specified material and torque, as illustrated by curve B, Fig.6.

Apart from the limit on allowable stress for the sliding speed at the rubbing contacts, as discussed in the next section.

Tooth Sliding Speed Limit

Following an extensive laboratory study of gear coupling performance by the U.S. Naval Boiler and Turbine Laboratory, in which both full scale and bench testing was carried out, Boylan, in 1966, showed that there appeared to be a critical tooth sliding speed, in the range 0.12 – 0.2 m/s, beyond which tooth fretting took a much more severe form now generally referred to as wormtracking (Ref.6). Most engineers have interpreted these values as meaning that 0.12 m/s is the maximum allowable sliding speed for continuous running, but that 0.2 m/s is permissible as a transient condition.

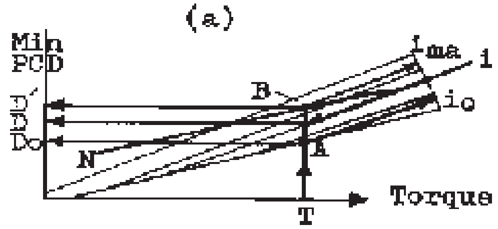
Using the continuous limit, the maximum allowable PCD can be plotted against angular misalignment for any given rotational speed. The result for a typical application is shown by curve C on Fig.6. The tooth shearing limit, curve D, has also been included for the same application in Fig.6, and can be seen not to be a critical condition. Fig.6 can now be seen to define the allowable combinations of coupling PCD and angular misalignment. It can be seen that the sleeve bursting limit in this case is not critical, and this would in fact be true for many applications. A useful practical deduction is that gear couplings can normally be expected to deteriorate progressively by fretting rather than to fail catastrophically by sleeve bursting, and this is borne out in practice and is an important advantage of gear couplings.

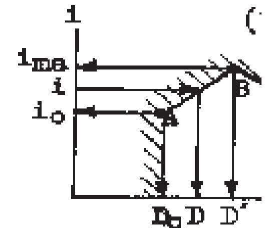
SELECTION OF MAIN COUPLING DIMENSIONS

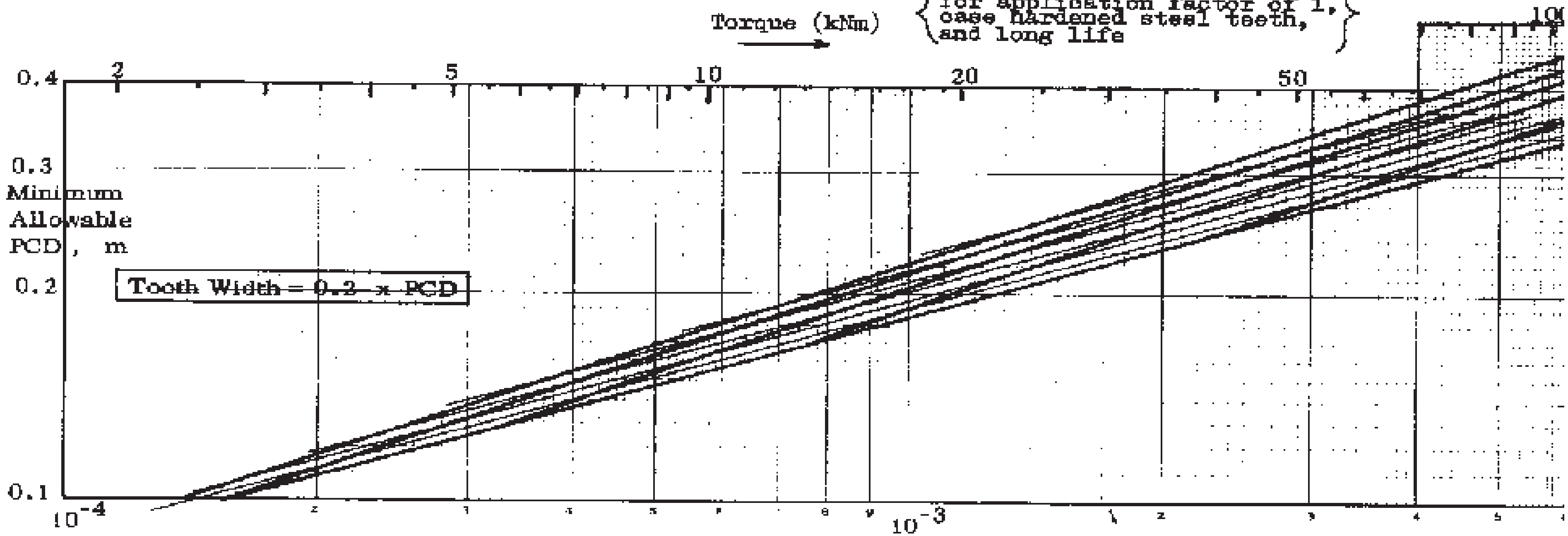
The need for low coupling mass to minimise unbalance and vibration problems implies the need for a small PCD. A design angular misalignment approaching the maximum allowable value also needs to be used, since otherwise the coupling is liable to be unduly long to cater for the specified offset misalignment. Consequently, a combination of PCD and angular misalignment near the top left corner of the allowable operating region to be selected, as indicated in Fig.6.

Fig.7 is a design graph based on the fact that the tooth fretting and sliding speed limits are critical, and assuming a tooth width to diameter ratio of 0.2. This value is at the top end of the range of current practice of about 0.12 – 0.2, but well below the range suggested by Dudley of about 0.25 – 0.4. At the very small angular misalignments which must be maintained in large high speed couplings, and in view of Dudley’s recommendations, it is suggested that the value of 0.2 may well be reasonable when designing for minimum coupling PCD and hence mass. If a larger PCD than the minimum has to be used for any reason, e.g. to reduce thrust transmitted by tooth friction or to accommodate a specified hub size, smaller values of tooth width should be used, since this will limit the backlash required, and the transverse moment generated at the meshes. Conversely if smaller tooth widths than 0.2 x D are used, a larger PCD than that indicated by Fig.7 will be needed.

USE OF FIG. 7 For a given torque (or torque parameter) T, move up to point A (see inset Fig. (a)) and read across to Do. This is the minimum allowable PCD and can be used with angular misalignment up to i_o, (see inset Figs. (a) and (b)). The maximum allowable angular misalignment, i_{ma}, may be found by moving up to point B (see (a)) on the speed line N for the application. For any chosen angle of misaligment, i, between i_o and i_{ma} the minimum allowable PCD, D, can be found by moving up to the appropriate angular misaligment line, i, and reading across to D







$\frac{\text{Torque x Application factor (Table 1)}}{\text{Allowable Contact Stress, } S'_c \; L_W}$ (m^3) (For S'_c see Table 4 and for L_W see Table 3)

FIG.7

SELECTION GRAPH FOR COUPLING PCD AND DESIGN ANGULAR MISALIGNMENT

It should be noted that the maximum allowable angular misalignment determined by Fig.7, is a total figure which has to cater for both angular and offset misalignment of the shaft ends. However the shaft angular misalignment will usually be kept to a few tenths of a mm/m or less in high speed machinery, so that the allowable angular misalignment to cater for shaft offset need only normally be restricted to a little less than the total maximum allowable figure. This reduced value then determines the combinations of minimum spacer length and maximum allowable shaft offset which can be used. However, several other factors need to be taken into consideration in choosing the spacer length, namely:

1. Assuming that a reversed hub coupling design is used to minimise overhung moment, and that a tree-piece-spacer design will be necessary for convenient coupling disassembly, the spacer length will need to be somewhat more than twice the PCD. The minimum length for a typical high speed spacer design for use with reversed hubs is shown in Fig.8.

2. Need to keep coupling spacer mass down, (although this is not in fact very sensitive to spacer length, as indicated for a typical design in Fig.8). Note that the maximum acceptable mass can be determined from the minimum acceptable value of the Conti-Barbaran parameter, which should be taken as 10 (Ref.7)

3. Need to keep the first bending critical speed of the coupling spacer at least 50% above the running range, for which the maximum allowable length between meshes in metres for a steel spacer cane be estimated conservatively from:

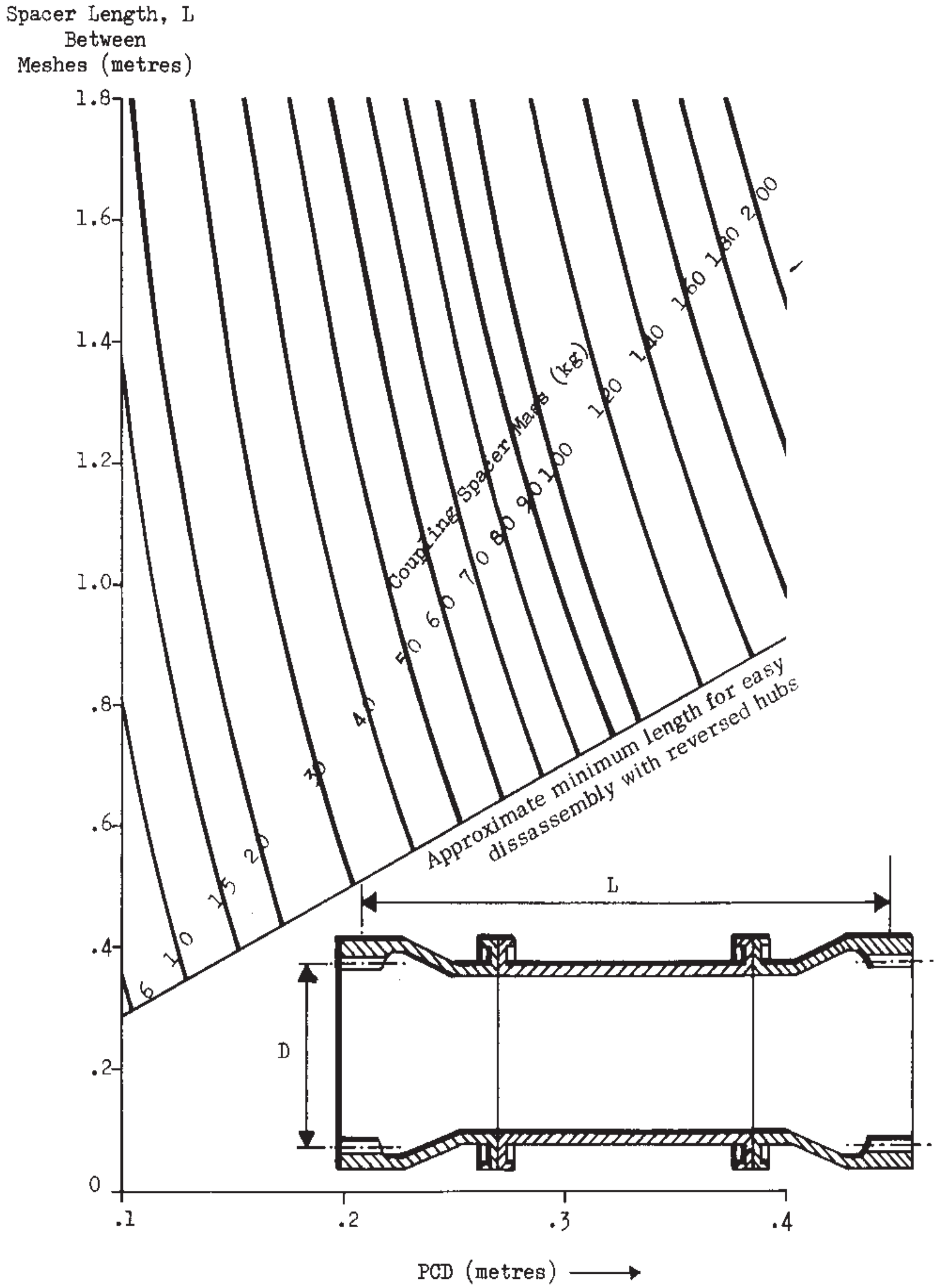
$$L_{max} = 340\sqrt{\frac{\text{Spacer tube internal diameter in metres}}{\text{Maximum running speed in rpm}}}$$

4. Desirability of saving floor space.

5. Desirability of keeping down the transverse forces generated by the coupling when misaligned, which reduce greater than linearly as coupling length is increased.

6. Desirability of avoiding spacer lengths which give a system torsional critical in the running range.

Based on the chosen value of design angular misalignment, the desirable amount of tooth crowning-if any-can also be assessed. There is both support and opposition for the use of crowning on low angular misalignment applications. If crowning is used, the contact stress will be higher on perfectly aligned teeth, but would tend to be less for a coupling misaligned to the design value. In practice, for low angular misalignment applications, the differences in terms of the direct effect on contact stress are probably slight, provided the amount of crowning does not exceed the value required for the maximum design angular misalignment. However, a crowned tooth would appear better from the point of view of any possible hydrodynamic lubrication effects, and perhaps for this reason slightly crowned specimens were found to wear marginally less than flat ones in lubricated fretting tests by McMath (Ref.8). Crowning also has the advantage that it somewhat reduces the tilting moment generated at a misaligned mesh, and this was considered the prime reason for introducing it by one manufacturer (Ref.9). Crowning has the additional merit, with nitrided teeth, that it should help to avoid



ESTIMATION OF SPACER MASS FOR A HIGH SPEED COUPLING DESIGN

FIG. 8

heavy edge contact which could lead to case exfoliation. Taken together, there is probably a small overall advantage for using very slight crowning.

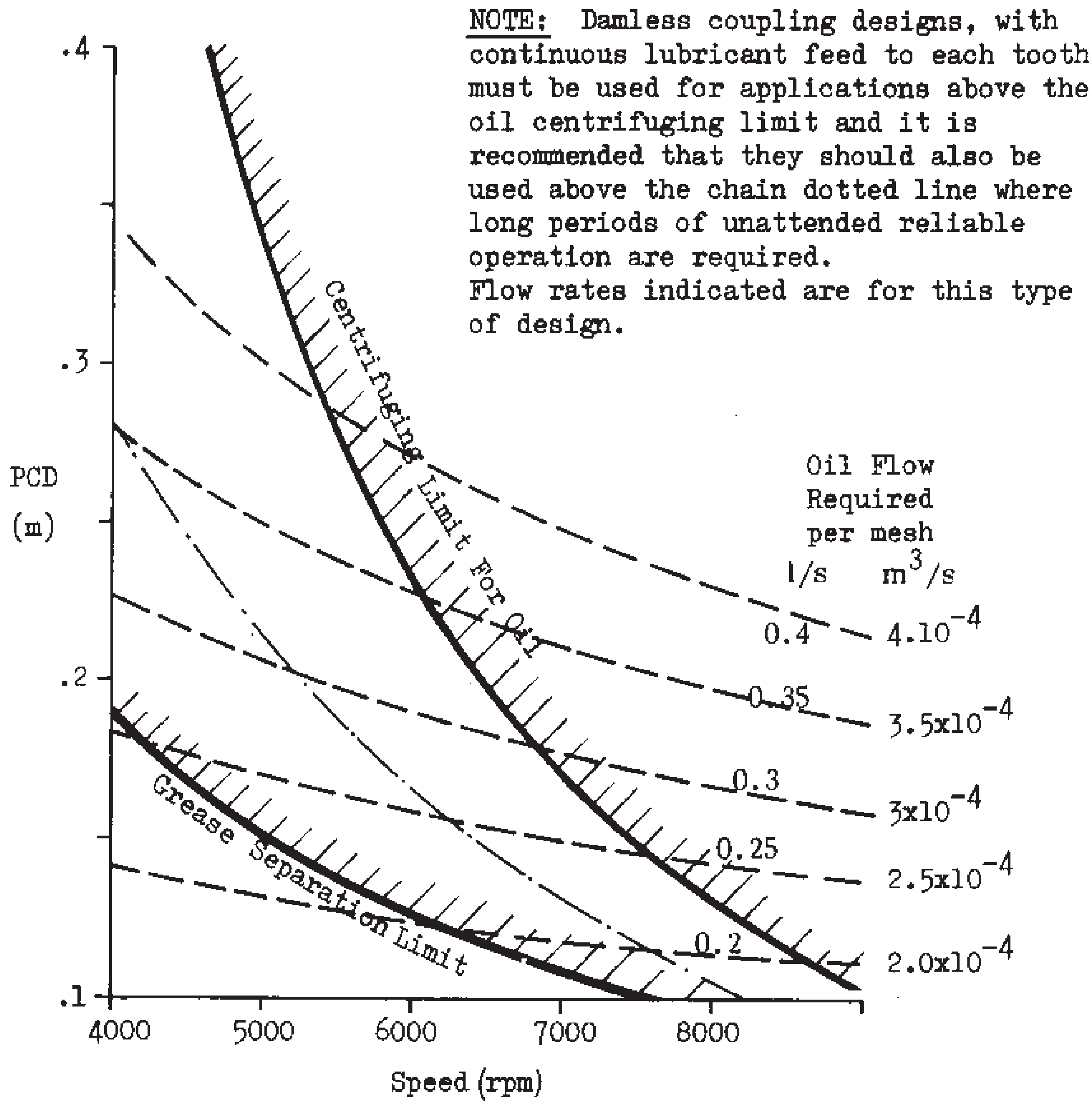
It is essential, however, that if crowning is used it should be the minimum necessary to accommodate the design angular misalignment. An appropriate amount of flank crowning, in terms of crown height, can be determined by choosing how near the tooth ends the nominal contact point should be for two teeth misaligned by the design maximum value, i. Assuming this distance to be one-fifth the tooth width, the crown height required is about (i/2.5) times the tooth width, i.e. .04 mm on a 50 mm wide tooth, if I = .002 rad.

If crowning is applied, it is essential that high precision is achieved, to avoid any adverse effect on tooth accuracy and pitch errors. If doubt exists as to the precision with which crowning can be applied, and for applications where the running angular misalignment can definitely be kept to .001 rad or below, at least as good performance is probably achievable with straight teeth.

LUBRICATION

Gear couplings need lubrication, and it has been estimated that 75% of failures are due to improper or insufficient lubrication. For long periods of unattended reliable operation at high speeds, oil must be used rather then grease, and a continuous feed to each tooth and a damless coupling design are essential. Grease separates at relatively low speed, as indicated in Fig. 9, and is susceptible to hardening, which can prevent it flowing to the teeth. Both oil and grease filled couplings are unsuited for long periods of operation because of seal unreliability, and gradual loss of charge. A continuous flow is also necessary at high speed, both for cooling and, in conjunction with a damless coupling design (see Fig.1), to prevent sludging problems which occur at high ‘g’ due to centrifuging out of additives, impurities and dirt in the oil. The approximate onset of a significant centrifuging effect is suggested by one manufacturer to correspond to a centrifugal acceleration $\left(\frac{D}{2} x\, \omega^2\right)$ of $4.5 \times 10^4 m/s^2$ (Ref.10) and this limit is indicated in Fig.9.

For long periods of unattended operation, a suggested size-speed limit above which the use of damless coupling designs with a continuous lubricant feed is recommended, is indicated by the chain dotted line in Fig.9. Guidance on the flow rates required with these designs is also given in Fig.9, based on the recommendations of one manufacturer (Ref.10). Damless coupling designs are liable to rapid deterioration in the event of a lubricant supply failure, and although this is probably of minor significance, since the oil feed will generally come from the bearing lubrication system, which will be provided with a low oil pressure trip, it does mean that careful design and installation of the piping to the coupling oil feed is essential. Also, with damless designs, lubrication may be impaired if a lot of wear is allowed to develop, due to over-easy escape of oil past the teeth, and this is a further reason for changing couplings before major wear develops.

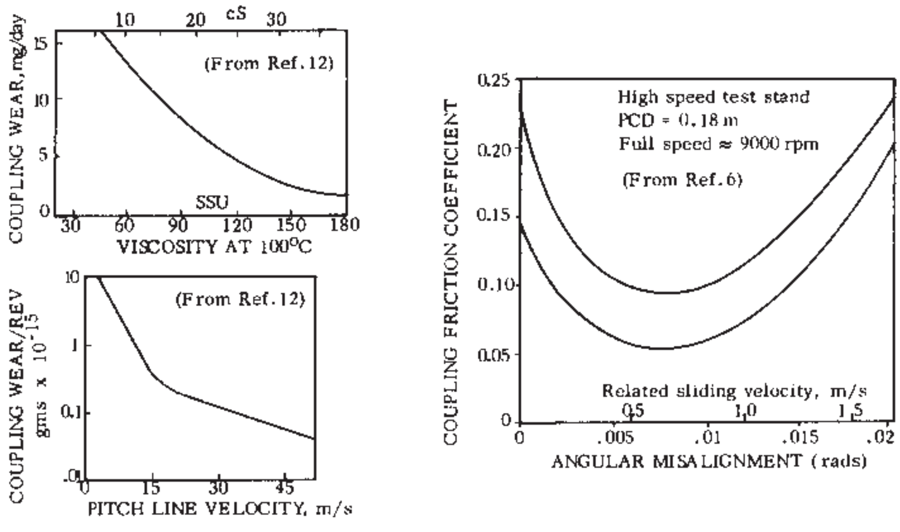


LUBRICATION REQUIREMENTS OF GEAR COUPLINGS FIG. 9

The type of oil used for lubricating gear couplings appears to have some effect on tooth wear. 90 EP and SAE 140 oils did well in laboratory tests aimed at simulating the coupling wear mechanism, (Ref.8), and other experimental data also indicates a beneficial effect of increased viscosity, (Ref.11), as shown in Fig. 10. In practice, however, the choice of oil for a continuous supply will normally be restricted to the limited range acceptable by adjacent parts of the system, since it will not generally be desirable to use an independent lubrication system for the couplings. The need for a high viscosity does emphasise the need for a cooled oil supply, however. The supply also requires filtration to as fine a value as the oil system will allow.

OIL FILM GENERATION IN COUPLINGS AND THE POTENTIAL FOR IMPROVED COUPLING PERFORMANCE

It will be apparent from the foregoing that existing design methods for low misalignment straight tooth gear couplings are largely empirical. Based on a more accurate representation of the basic actions involved, better methods of assessment could almost certainly be developed, and it will also be suggested in the following that there may be potential for development of gear couplings of improved design and performance. Tooth fretting and wormtracking are the main limiting conditions on coupling performance, and empirical values for the maximum contact pressure for different materials, together with Boylans's maximum sliding speed, are currently used as the criteria for obtaining acceptably wear free designs. However, the Boylan limiting sliding speed, although valuable in the absence of anything better as a guide, is probably not a fundamental and generally applicable limit. For example, in rolling mill drives, satisfactory performance is obtained with considerably higher sliding speeds, approaching 0.5 m/sec. There is also test evidence available (Ref.12) that shows tooth wear rates actually reducing with increasing coupling rpm and with increasing lubricant viscosity. These effects are shown in Fig. 10. Fig. 10 also reproduces data from Ref. 6 for the measured friction coefficient as a function of misalignment and hence sliding speed, on a high speed test stand for which the maximum allowable angular misalignment corresponding to the Boylan sliding speed limit was apparently only 8' (.0023 rads). It can be seen that the friction coefficient continues to decrease with increase of sliding speed, up to at least four times the Boylan sliding speed limit.



EVIDENCE OF HYDRODYNAMIC EFFECTS IN GEAR COUPLINGS FIG.10

All this data, as well as the practical experience that couplings can show almost negligible wear after several years in service, suggests the existence of pronounced beneficial hydrodynamic effects which help to separate the tooth surfaces as sliding speed and viscosity increase.

Clearly, an estimate of the minimum oil film thickness between the teeth should be relevant to the performance of couplings, just as it is in other tribological components. In recent years, following the establishment of formulae for calculating film thickness at rolling/sliding contacts of various configurations, the importance of film thickness to surface roughness ration has become widely recognized, and is now being incorporated into design assessment procedures for many common components, such as gears and rolling bearings. The film thickness between the teeth of a coupling in a typical application will be estimated by way of example.

The example which will be considered is a 0.25m PCD coupling, with 60 crowned teeth, 75% full depth and 20º pressure angle, running at 6,000 rpm. A design angular misalignment pf 0.003 rads will be assumed, with an appropriate crowning radius in the normal plane of 4m. With this radius, heaviest contact will be made at an axial distance of 4 x 003 =12mm, from the hub centre plane, which could be accommodated by a tooth of reasonable width.

Let us now consider the conditions of contact between two teeth when at the position of maximum relative tilt, i.e. when carrying maximum load. The contact is equivalent to that between a plane and a cylinder of 4m radius, and height equal to the working length of flank, 6.7 mm. The relative movement between the external and internal teeth at this stage of the cycle is pure sliding without any rolling, and this provides an oil film generation mechanism. The oil film thickness can be calculated using the expression of Dowson & Higginson, (Ref.13) which can be written as:

$$\frac{h}{R_{cu}} = \frac{1.6\ G^{0.6}\left(\dfrac{\eta_o\ u_{ent}}{E'R_{cu}}\right)^{0.7}}{\left(\dfrac{w}{E'\ R_{cu}}\right)^{0.13}}$$

where:

h	=	minimum film thickness in the contact,
R_{cu}	=	radius of curvature of the equivalent cylinder,
η_0	=	dynamic viscosity of oil at the bulk temperature of the teeth,
u_{ent}	=	entraining velocity = average velocity of the two mating surfaces relative to the contact point,
E’	=	equivalent Young’s modulus of surfaces,
	=	$\frac{E}{.91} = \frac{200\ \times 10^{9}}{.91}\ \frac{N}{m^{2}}$, for steel surfaces ,
G	=	5,000 for steel surfaces lubricated by mineral oil,
W	=	load per unit length of contact.

Based on the standard relationships for cylindrical Hertzian contact (see for example, Ref.14) the load per inch of contact can be replaced by

$$w = \left(\frac{p_{max}}{.399}\right)^{2}\ \frac{Rcu}{E'}$$

where p_{max} is the minimum contact pressure at the centre of the contact area.

Also, the entraining velocity in pure sliding velocity in pure sliding equals half the sliding velocity, V_s. Thus, for sliding steel surfaces lubricated by mineral oil, the film thickness can be calculated from:

$$h = \frac{0.00131\left(\eta_o V_s\right)^{0.7} R_{cu}^{0.3}}{p_{max}^{0.26}}$$

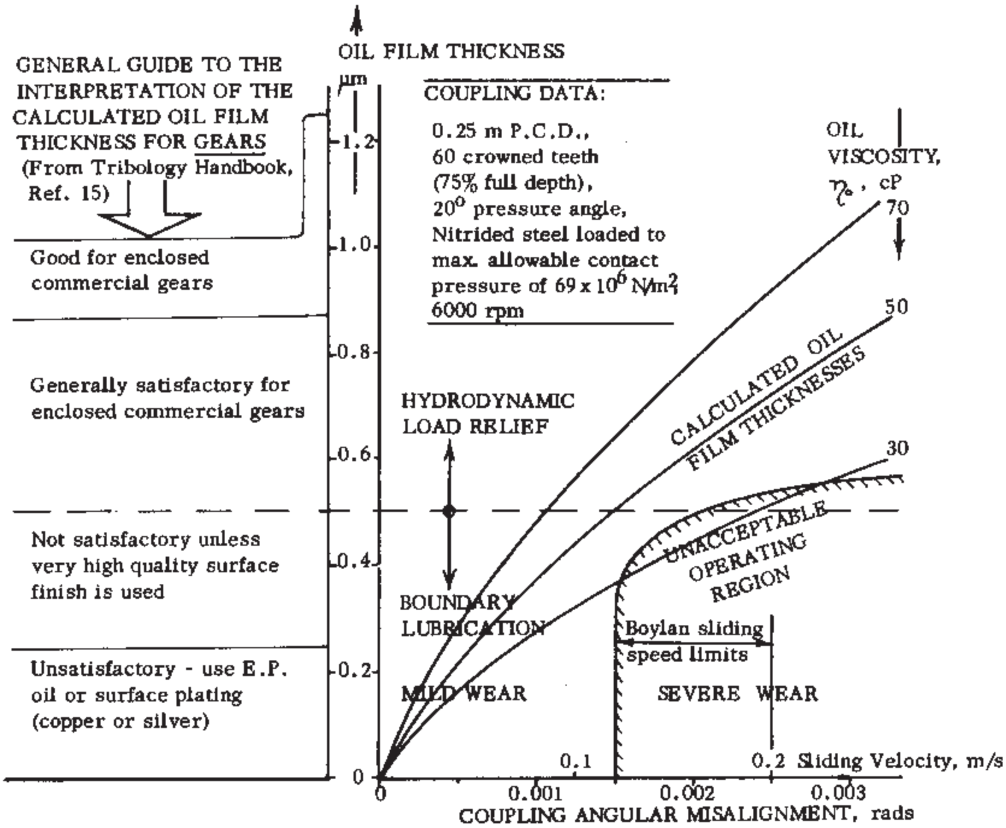
Now p_{max} can be taken as the maximum permissible contact pressure for the surfaces which, for crowned teeth, Dudley recommends being taken as four times his values for straight teeth, which are given in the Appendix by S'cLw. Thus the max permissible contact pressure for case hardened steel would be 69 x 10^6 N/m^2. Also R_{cu} is simply the tooth crowning radius in the normal plane, i.e. 4m in the present example. Therefore, in this example:

$$h = 0.0000182\left(\eta_o V_s\right)^{0.7}$$

A typical oil viscosity might be 0.05 Ns/m^2 (50 cP) corresponding to SAE 20 at 42°C, and in the present example the sliding velocity would be: 0.24 (i/ .003) m/s. Substituting these values gives:

$$h = 0.82 \left(\frac{i(\text{rads})}{.003} \times \frac{\eta_o(\text{cP})}{50}\right)^{0.7}$$

Thus, at the design angular misalignment, with a 50cP oil viscosity, the film thickness would be about 0.8 μm and would increase with angular misalignment and viscosity, as indicated in Fig.11.



<u>CALCULATED OIL FILM THICKNESS IN A TYPICAL COUPLING WITH SUGGESTED REGIONS OF HYDRODYNAMIC LOAD RELIEF AND UNACCEPTABLE OPERATION.</u>

<u>FIG.11</u>

By comparison with gears, satisfactory operation could be expected with film thicknesses above about 0.5 μm; whereas little hydrodynamic load relied could be expected with film thicknesses less than about 0.5 μm and boundary lubrication would prevail, (Ref.15). In boundary lubricated contact the friction coefficient is relatively high (around 0.15) and in the absence of load relied by hydrodynamic effects, experience suggests that there will be a transition to severe wear above some relatively low limit of sliding speed, associated with the generation of appreciable frictionally generated temperature. For steel surfaces lubricated by mineral oil it seems likely that the limit is that found by Boylan, i.e. about 0.12 m/sec.

However, the implication of Fig.11 is that either the velocity must lie below 0.12 m/s, or the film thickness must exceed a value of about 0.5 μm, if a low coupling wear rate is to be achieved. This defines a rectangular 'danger' area on the film thickness/sliding speed graph; in reality it is likely that the boundary would be curved as indicated diagrammatically in Fig.11.

This possibility of exceeding the Boylan limit, perhaps appreciably in favourable circumstances, therefore seems to accord with practical experience and is clearly important and worth further study. The following conclusions and proposals for further work are suggested:

1. Hydrodynamic effects may be capable of extending the misalignment limits of crowned tooth couplings to several times their present restrictive levels.

2. These beneficial effects would not be obtained with straight male teeth at the end of the coupling where the male teeth are driven. This suggests the superiority of crowned teeth or hermaphrodite spacers.

3. A study of the effects of crowning on load distribution, maximum tooth contact pressure, minimum oil film thickness, tilting moment, and backlash, would be worthwhile, and should be aimed at defining an optimum amount of crowning for a given design angular misalignment. If, as is likely, the amount of crowning is very small for low misalignment applications, the method of accurately producing such crowns should be reviewed to establish the best method.

4. The effect on oil film thickness of the cyclic relative movement pattern between the teeth should be studied. The entraining velocity varies from a value far in excess of the sliding sped down to zero shortly before the maximum load point. Squeeze film action will limit the minimum film thickness occurring at the point of zero entraining velocity, and the minimum cycle film thickness may not differ greatly from that calculated above on the basis of an entraining velocity equal to half the sliding speed; but this needs to be checked.

5. The use of a sufficiently thick oil should ensure hydrodynamic load relief before the sliding velocity reaches a critical level. In the example considered earlier, it appears that an SAE 20 oil would have been adequate, provided the tooth temperature was kept below about 40°C. The major extension of coupling misalignment capacity which appears possible with a sufficiently thick oil suggests that serious consideration to the following should be given in any particular case:

a. Increasing the oil flow rate to the coupling to maintain a low temperature rise of the meshes.
b. Use of the thickest grade of oil which can be accepted by all the lubricated components in a common lubrication system.
c. Cooling the oil supply.
d. Ensuring that the ambient temperature inside any casing surrounding the coupling is kept down by venting the casing.
e. Air cooling the meshes, for example by means of a finned, light alloy, shrunk-on ring around the OD of each coupling mesh.
f. Use of a separate lubrication system for the coupling with a much thicker oil grade, and boundary lubrication additives.

6. The use of smoother tooth surface finishes, to permit hydrodynamic load relief even with relatively thin oil films, should be considered.

7. Improved assessment procedures for couplings should be developed, based on the methods of Renzo et al (Ref.2) and Airapetov et al, (Ref.4) which reflect more accurately the actual patterns of tooth contact and stressing and should enable better optimization of tooth number and geometry.

8. Development of new coupling tooth materials, capable of accepting higher sliding speeds without transition to heavy wear in boundary lubrication, should be attempted. Some of the coatings and treatments which have appeared in recent years, particularly those providing reduced friction coefficient in conjunction with reasonable hardness, would be worth testing under conditions closely simulating actual tooth contact and movement patterns.

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APPENDIX – SUMARY OF DESIGN DATA FOR HIGH POWER & SPEED COUPLINGS

Notation

b	Width of shorter teeth at a mesh, m
b_{eff}	Maximum effective tooth width (see Fig.12)m
B	= b/D
D	Pitch circle diameter (PCD), m
h	Common tooth depth, m
i	Angular misalignment at a mesh, rad
k_w	= t_w/D
k_a	Application factor (see Table 2)
k_m	Load distribution factor (see Table 2)
k_s	Load sharing factor=proportion of teeth assumed to carry load-½ for normal, and $\frac{1}{3}$ for poor manufacture (Ref.5)
L_f	Fatigue life factor (see Table 3)
L_w	Wear life factor (see Table 3)
n	Number of teeth
N	Rotational speed, rpm
R	Pitch circle radius, m
S_t	Combined bursting stress at root of sleeve teeth, N/m^2
S_{cf}	Bursting stress due to centrifugal force, N/m^2
S_s	Shear stress in teeth at pitch circle, N/m^2
S_{tf}	Bursting stress due to torque induced tooth force, N/m^2
S'_c	Allowable compressive stress (see Table 4) N/m^2
S'_s	Allowable shear stress (see Table 4) N/m^2
S'_t	Allowable tensile stress (see Table 4) N/m^2
t_c	Chordal tooth thickness at pitch line, m
t_w	Radial thickness of the sleeve at a mesh, m
T	Torque, Nm
V_s	Tooth sliding speed, m/s
V_r	Sleeve peripheral speed, m/s
Y	Lewis factor for sleeve teeth
Ø	Pressure angle, °

Summary of Design Equations (based principally on data from Ref.5).

1. Tooth form: shaved or ground teeth with:

   20° pressure angle, 75% full depth teeth.
   or 25° pressure angle, 70% full depth teeth.

   Former gives more flank area, and lower bursting stress, and is suggested to be the preferred form. Latter gives greater centring action.

2. Tooth number, n: is not critical, but should be even. Suggested values are:

D	0.1	0.2	0.3
n	48	56	60

3. Sleeve thickness ratio, $k_w = t^w/D$. Typical range is 0.05 to 0.1. Suggested value for initial assessment is $k_w = .08$.

4. Tooth width ratio B = b/d.

   Choice of B depends on several factors, and is difficult to determine logically without more understanding of tooth stresses in misaligned couplings. Increasing B reduces tooth stresses in a perfectly aligned coupling, but in a misaligned coupling the tooth load is concentrated at one end, and additional tooth width beyond some value will not assist in bearing the tooth load at all; increasing B also increases the required tooth clearance and backlash, as well as the tilting moment.

   The design angular misalignment will clearly have an important influence, and larger B values will be appropriate when the design misalignment angle is small. Dudley (Ref.5) suggests values of 0.25 to 0.4 with the latter value implied as being appropriate for a misalignment of about 1mm/m. Typical current practice is for B=0.12 to 0.2, with the lower values probably intended for somewhat larger misalignment angles.

   Where the misalignment angle must be kept down to about 1mm /m, and the minimum PCD and hence coupling weight is sought, it is therefore considered reasonable to be somewhat influenced by Dudley's recommendations and to design for a B value at the top end of the range of current practice. The suggested value is: B=0.2; b_{eff}/D may then be taken as 0.2 or as indicated in Fig.1, whichever is smaller.

5 Centrifugal bursting stress in sleeve.

$$S_{cf} \approx 7400\ V_r^2, \text{ where } V_r = \frac{DN}{60}\left(1 + \frac{\pi}{2n} + 2k_w\right)$$

$$\therefore S_{cf} = 28.6\ D^2N^2 \text{ for } n = 56 \text{ and } k_w = .08.$$

6. Torque induced bursting stress in sleeve.

$$S_{tf} = \frac{T\ \tan\Phi}{D^2 k_w b} + \frac{2T}{K_2 D^2 b_{eff} Y}$$

component due to radial tooth forces — component due to tooth bending under tangential tooth force

For precision couplings, $K_s = ½$. Also $Y \approx 1.5$.

$$\therefore S_{tf} = \frac{T}{D^3}\left[7.24 + 13.33\left(\frac{0.2D}{b_{eff}/D}\right)\right] \text{ for } Ø = 20^\circ \text{ and } k_w = .08.$$

7. Combined bursting stress in sleeve.

   $S_t = S_{tf}\ K_a K_m + S_{cf}$. For satisfactory performance, $S_T < 0.3\ S'_t$, see Table 4.

   Note: S_{tf} will fluctuate at twice shaft frequency in a misaligned coupling, whilst S_{cf} will be steady.

For the purposes of a conservative design method, S_t is therefore compared with a high cycle fatigue tensile strength under fluctuating stress, and S_{cf} is factored by ½ as indicated above, to convert it into an equivalent fatigue stress. The factor 0.3 is the fatigue life factor obtained from Table 3 for high cycle fluctuating stress.

8. Tooth flank pressure (for straight or near-straight teeth)

$$S_c = \frac{2TK_m}{Dnb_{eff}h} \approx \frac{T}{D_3}\left[6.37K_m\left(\frac{0.2D}{b_{eff}}\right)\right]$$

For satisfactory performance, $S_c \times K_a < S'_c \times 0.5$. For S'_c, see Table 4. The factor 0.5 is the wear life factor obtained from Table 3 for very high cycle rubbing (10^{10} cycles).

9. Tooth shearing stress (shearing assumed at pitch circle)

$$S_s = \frac{2TK_m}{Dnb_{eff}K_s t_c} \approx \frac{T}{D^3}\left[12.7\,K_m\left(\frac{0.2D}{b_{eff}}\right)\right]$$ (For precision couplings, $K_s = ½$)

For satisfactory performance, $S_s \times K_a \le S'_s \times 0.3$. For S'_s see Table 4. The factor 0.3 is the fatigue life factor obtained from Table 3 for high cycle stress.

From 8 and 9 above it can be deduced that :

$$\frac{\text{Safety factor on shear stress}}{\text{Safety factor on flank pressure}} = 0.3 \times \frac{S'_s}{S'_c}$$ = 3 to 5, i.e. shear stress in teeth is not critical.

10. Tooth sliding speed

$V_s = \frac{\pi DNi}{60}$. For satisfactory performance: $V_s \le 0.12$ m/s for continuous operation

and $V_s \le 0.2$ m/s for transient operation (Ref.6).

Power source	Application Factor, K_a, with a uniform load
Turbine	1
Motor through gears	1.25

TABLE 1

Angular misalignment, i (rads)	Load distribution factor, K_m, with face width, b, equal to:			
	0.0125m	0.025m	0.05m	0.1m
0.001	1	1	1	1.5
0.002	1	1	1.5	2.0
0.004	1	1.5	2.0	2.5
0.008	1.5	2.0	2.5	3.0

TABLE 2

Number of cycles or revolutions	Fatigue Life Factor, L_f, with :			Wear Life Factor, L_w
	Fully reversed stress	Unidirectional stress	Fluctuating stress*	
10^3	1.8	1.8	-	-
10^4	1.0	1.0	1.1	4.0
10^5	0.4	0.5	0.7	2.8
10^6	0.3	0.4	0.6	2.0
10^7	0.2	0.3	0.5	1.4
10^8	-	-	0.4	1.0
10^9	-	-	0.35	0.7
10^{10}	-	-	0.3	0.5

*Typical of twice per cycle tooth stress variation in a misaligned gear coupling

TABLE 3

Material	Hardness		Allowable Stresses in MN / m^2		
	Brinell	Rockwell C	S'_t	S'_c	S'_s
Steel	160-200	-	150	10	140
Steel	230-260	-	220	14	210
Steel	302-351	33-38	310	21	280
Surface hardened steel	-	48-53	310	28	280
Case hardened steel	-	58-63	380	34.5	345
Through hardened steel (aircraft quality)	-	42-46	345	-	310

TABLE 4

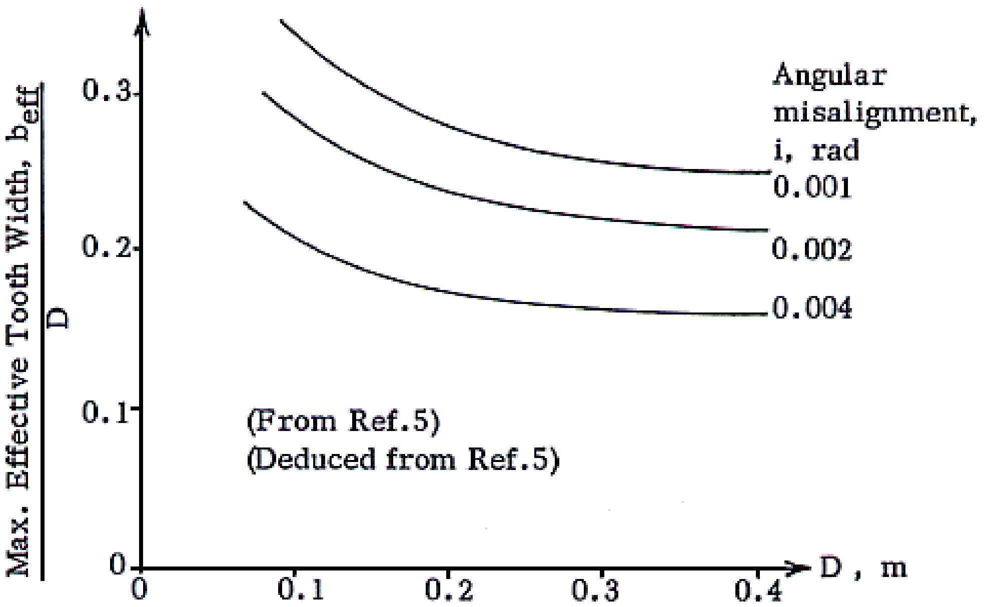

FIGURE 12