

## Discussion on Operating Experience

### DISCUSSION SUMMARY by B.J. Woodley

A large part of this discussion on operating experience considered the problems of obtaining an adequate connection between the machine shaft and the flexible coupling hub in high power, high speed applications. A user introduced the subject to the Conference by giving an example of a 30,000 Hp turbine which had been completely destroyed due to a fracture of the end of its shaft caused by an interaction between the hub of the flexible coupling and the shaft. He stated that his company had had two such experiences and during 1976 they had also suffered six near catastrophes from the same cause which, luckily, had been avoided by the discovery of cracks in shafts when the machines had been stopped for other reasons. The problems had arisen on hub joints with a shrink fit and a key-way and no warning had been obtained from vibration monitoring. The in-depth studies carried out by the company in an attempt to solve the problem were then summarised.

Originally it had been thought that the sole cause of the failure was fretting corrosion giving rise to stress concentrations, from one of which a hair-line crack would initiate and then propagate to give shaft failure. However, this is no longer thought to be the only cause as, although it has been possible to prevent the fretting corrosion by going to an interference fit of 1 thou/in. on a 30,000 Hp machine with a 6 inch diameter shaft, shaft end cracking has again occurred. It was also noted that some companies operated quite satisfactorily with relatively high levels of fretting corrosion and this again tended to confirm that fretting corrosion is not the real cause of the problem. The next stage of investigation involved a study of the real stress distribution around the key-ways and the impression obtained at the current stage is that even the most conservative calculation methods are not sufficient. It was noted that the cracking always occurs at the bearing end of the key-way near the rounded end of the key slot and that this is where three stress concentration factors combine. These three are caused by (1) shrink fits, (2) key load due to torque, and (3) dynamic bending moments due to gear coupling tooth contact. Therefore, a three dimensional technique is required to analyse the stresses at this point and a programme called ENSIS is now available to do this. However, the user's current policy is to do away with keys and a method of brazing half keys into the slots was outlined as a useful way of filling the key-ways on existing machines. The coupling hubs can then be hydraulically fitted to the machine shafts using a shrink fit of 2 thou/inch and following the SKF guidelines. The user felt that a flange forged on the shaft is the best system, but different shaft sealing systems to those currently available are required to make this a practicable method.

Another user agreed that although it is not currently possible to prove theoretically that shrunk-on fits have an even greater stress concentration effect than usually allowed for in design, shafts do fracture from this cause and so it must indeed be the case. It was mentioned that because of this effect the rotors on large 500 MW generating sets are now solid forged or welded rather than using shrunk-on fits. It was also noted that in the chemical industry, couplings are often just fitted cold on to taper shafts because of the need for fast overhaul periods, and that this is not really satisfactory as the fretting which normally occurs, particularly on the non-drive side of the key, means that reclamation

lapping must be carried out every few years to obtain a perfect fit again. This causes a ridge on the shaft and so the corner must be taken off the coupling hub to permit it to be drawn on to the shaft. It was also mentioned that some new designs are now using splines with radial registers each side of the spline, rather than the conventional tapers. One delegate asked if there had been any experience of high shrink fits having a negative damping effect and so lowering the critical speed, but the view was expressed that negative damping was only a problem in a self-excited whirl situation and so should not be a major obstacle to the further use of high interference fits.

A manufacturer pointed out that on low speed plant, carbon steel shafts are used and the shaft is relatively large to carry the stresses and so keys can be large and tend to be satisfactory. On the other hand, high speed plant use alloy steel shafts which can carry high stresses and so can be smaller giving proportionally smaller keys. At the same time, speeds are high so the hub can grow under the centrifugal stress giving a significant loss of interference fit. Therefore, on high speed shafts the manufacturer feels that hubs should be hydraulically fitted with about 2 thou/inch interference fit, rather than use keys and key-ways and it was also noted that many people also use a slight taper, making it easier to get the hub off. However, although it was generally agreed that hydraulically fitted hubs are currently the best available method of mounting on high speed shafts, it was also very clear that the most important current practical problem is the interaction which can occur at the shaft to hub connection. At the present time it was pointed out that it is essential to check for excessive stresses in the shaft ends before a machine is put into operation and to examine it at every shutdown. It was noted that the turbine shaft end tends to give the most problems as the coupling size is usually specified by the compressor manufacturer who can use better quality materials as he has just to make a relatively simple cylindrical shaft with shrink fitted and keyed discs, whereas the turbine manufacturer has a different problem which means he cannot use such a good shaft material, but usually has to restrict his shaft size to fit the specified coupling.

There was also considerable discussion on what was a suitable coefficient of friction to assume at the teeth of gear couplings when designing thrust bearings, as the use of too large a value of coefficient of friction leads to large bearings with high power losses. It was noted that the American Petroleum Institute, in API 617, specify thrust loads based on a coefficient of friction of 0.25 and also suggest that thrust bearings should be loaded to no more than 50% of the bearing manufacturer's rating. It was asked whether this American recommendation is relevant in European designs and a user pointed out that a major general difference between American and European manufacturers is that the former tend to use dammed designs which can give sludging problems, whereas the latter use damless couplings which do not seem to suffer such difficulties. It was therefore asked whether the dammed couplings and, therefore, if damless designs are specified, whether a value of 0.2, or even 0.15, might be acceptable. A coupling manufacturer pointed out that API is basically a user organisation which is trying to protect users and is, therefore, not really concerned if turbines and compressors have extra large thrust bearings, but, unfortunately, they are forcing machine and coupling manufacturers to oversize equipment to a point where it is no longer economical to manufacture. He then proceeded to outline the extensive tests they have recently carried out in this area, a full report of which is due to be published next year. It was said that

during the test couplings are operated at 100% - 200% load for 48 hours continuously in order to ensure oil is squeezed out as in practical situations, they are then forced to move at various axial rates commensurate with those which take place during thermal expansion of shafts, and even at zero misalignment – the worst case – the values of coefficient of friction are always less than 0.16. It was also noted that the tests have been carried out with badly worn teeth as well as new couplings and the test rig also moves alternately in each axial direction and combinations of couplings with gear teeth at one end a flexible diaphragm at the other are used to eliminate interactions. The only way a high coefficient of friction could be obtained in the test rig was by cutting the lubricant flow which should never be allowed to occur in real situations and therefore the manufacturer could not understand how the high values of coefficient of friction of 0.25 were arrived at.

If the test rig was accurately simulating real gear couplings in service, one delegate could not understand why thrust bearings have failed previously when designed using a coupling friction coefficient of 0.15 and, in reply, a user pointed out that many of the bad earlier experiences of failing thrust bearings was caused by imbedding of the teeth which can occur when relatively soft tooth materials are used at high duty and, although this is not really friction, these experiences were probably the reason why values of friction coefficient of 0.3 were arrived at. The user noted that current practice is now to insist on nitrided and shaved tooth surfaces for high power, high speed gear couplings and another speaker stated that his company's present policy is to specify a value of friction coefficient of 0.15 and at the same time limit the load on the thrust bearing to a specific value of contact pressure. It was generally agreed that there is a real need to decide what coefficient of friction should be used when designing thrust bearings in systems using high quality gear couplings with nitrided teeth and, although no consensus of opinion emerged during the discussion on what the actual value should be, there was a strong feeling that the API value of 0.25 may be unnecessarily conservative especially with damless designs of gear coupling in the light of European experience.

Considerable discussion also took place about the capability of the various types of flexible coupling to accommodate overloads due to machine problems, such as pump seizures, liquid slugs getting into compressors, generator short circuits and mal-synchronisations. It was claimed that the contoured diaphragm coupling can survive pump seizures and one case was quoted where such a coupling even survived two consecutive pump seizures. However, another delegate noted that seizures of turbine driven pumps only cause relatively slow and relatively small overloads because the turbine rotor inertia can only give rise to about twice its full load torque. The real question was whether such a coupling could survive alternator short circuits which can give overloads of the order of nine times full load torque. A manufacturer of such couplings replied that they can be designed for short circuit faults, i.e. to take many times full load torque and that, recently, they had an experience in which a transformer was connected 180° out of phase and the coupling showed no distress afterwards. Another speaker noted that gear couplings always live through pump seizures and gave an example of an instantaneous stoppage of a compressor driven by a synchronous motor caused by an operator allowing liquid to get into the compressor. Although the incident caused both the double keys at the hub shaft connection to shear and the hub bore to be scored until the motor stopped, the gear coupling teeth were still in perfect condition and the coupling was re-used after the hub had been rebored.

The robustness of gear couplings was emphasised and it was noted that they usually even survive generator short circuits and mal-synchronisations.

A manufacturer of contoured disc couplings pointed out that the disc is usually designed to be the weakest link of the system so that if a seizure does occur then the diaphragm will fail and no expensive shaft damage will occur. Although a guard around the disc should stop the coupling flailing, many delegates were worried about the rapid disconnection of a turbine which could occur if the diaphragm failed. It was thought that in such a situation it would be likely that the common forms of overspeed trip would be insufficient and in marine applications it was noted that the Lloyds Register does not allow the possibility of a steam turbine disconnecting. Although manufacturers noted that contoured disc couplings and multiple membrane couplings can be designed to give adequate strength to survive overloads by making them bigger or thicker and that overload facilities such as collars or teeth can be designed so that they can take up the overload condition before the membrane goes past its yield point, a user emphasised that the real problem is the difficulty in deciding what transient overload conditions may take place so that these can be specified at the design stage. A manufacturer agreed that a major problem is the lack of information on 'off-design' considerations such as peak loading and surging in compressors as the designer cannot just oversize the coupling to take account of anything that might happen because this gives a high coupling weight and hence the possibility of lateral vibration problems. He stated that the designer therefore has to make a judgment based on as much data as possible and, although this was generally agreed, a user in the chemical industry pointed out the extreme difficulty of obtaining this data by instrumenting machines, because little of the electronic measuring equipment, which can give the required information, is made intrinsically safe. It was also noted that the electronic measuring equipment must also operate continuously for two years without adjustment in a fairly arduous machine environment.

One user noted that his experience of contoured disc couplings had been very satisfactory, although, compared to gear couplings, their use in his company had been very restricted – just 40,000 total operating hours on two simple shaft systems. He did mention that one significant factor has been the much reduced hub fretting with contoured disc couplings, whilst gear couplings require significant reclamation by lapping at the hub/shaft connection during every two year shutdown. However, it was stressed that it was too early to know whether this was going to be a general rule. Although several comments were made about the relatively limited axial displacement capacity of contoured disc flexible couplings, a manufacturer noted that they are developing new types to improve their capability. A new type of disc with a wavy form profile or 'mid-plane curvature' which has linear spring rate in addition to higher axial displacement capacity was said to have been operating satisfactorily in the field for the last three years. Another delegate mentioned that they are currently evaluating the potential of using contoured disc couplings and asked what type of surface inspection technique can be used to check the discs during maintenance turn-rounds. A manufacturer replied that although the discs are painted, should a crack occur it can be seen with a naked eye quite easily. Another user asked whether vibration monitoring could indicate the exact nature of the coupling failure and, although the vibration level was said to be a good warning of failure, it was noted that a considerable amount of vibration analysis using the latest powerful equipment would be required to define the problem exactly.

Other subjects mentioned in the discussion were also repeated in written contributions from delegates, which are given below, together with the replies from the authors of the papers.

P.W. SANDS Woodall-Duckham Ltd., England)

In relating instances of gear coupling failures, Mr. Andrews described an occasion when a serious failure occurred, associated with sludging at which fracture of the gear teeth took place. It was significant, as Mr. Andrews stated, that the deterioration was not recorded by the bearing monitoring equipment.

Whereas the gear failure is serious, perhaps more worrying overall is the doubt concerning the capability of the monitoring equipment to detect the deterioration and whether other couplings might be failing in service, though undetected.

It would be useful to know what measures Mr. Andrews took to satisfy himself regarding the integrity of the monitoring equipment and whether he retains his confidence in assessing the condition of the coupling by this method.

AUTHOR'S REPLY : F.A. ANDREWS (I.C.I. Ltd., England)

The examples of coupling sleeve fracture (Fig. 4) and heavy imbedding (Figs. 7A and 7B) were included in the paper in order to emphasise that extensive damage could occur without early warning from vibration monitoring equipment. The strength of both couplings had been substantially reduced and it is probable that catastrophic failures have resulted from sudden shock loading which had been induced by heavy surging.

This underlines the necessity for highly reliable vibration monitoring equipment to be continuously operated with automatic tripping of the machine at some suitably chosen danger level. In order to achieve the required level of reliability, an increasing use of voting systems between the individual transducer inputs is likely to be required in future vibration monitoring systems.

An alternative method of protection would be to monitor relative circumferential displacement between either the associated shaft ends or the spool piece and each shaft end. Such measurements are possible, but at this time no intrinsically safe commercial equipment is known to be available to carry out this task.

R. BOOLE (The Wellman Bibby Co. Ltd., England)

On page 7 it is stated that site balancing of gear couplings is not generally necessary; in view of CEGB's preference for straight teeth, and Boylan's comment that there are as many unbalance indications as number of teeth, I would be interested to know how hubs and sleeves are mutually located diametrically, and the designed clearance between hub and sleeve teeth. When couplings were grease lubricated, which was the major point from which the grease escaped – or did the oil evaporate, leaving the soap behind?

I wonder why CEGB favours straight teeth as even slight misalignment can give rise to intense pressure at the teeth ends; was this considered as a possible cause of the Cottam failures on page F2-15?

One wonders if Boylan's limiting rubbing velocity of 5 in per sec. Is relevant to straight teeth as there is much in his paper to suggest (though he doesn't specifically say so) that he was dealing with barreled teeth for which contact conditions would be very different. Only slight barrelling (readily applied by hobbing machines equipped for crowning) would be required to shift contact points in from the teeth ends with the consequent benefits of reduced Hertz stress, reduced teeth flank clearance and more teeth sharing the load; if desired wider misalignment limits would also be permitted.

The good performance of the majority of straight tooth couplings is a tribute to the high standard of CEGB alignment procedures, but doesn't the Cottam experience suggest rather that barreling is positively beneficial?

AUTHOR'S REPLY: F.O.J. OTWAY (C.E.G.B., England)

Mr. Boole has made a number of comments on my paper.

Regarding site balancing my statement should be read in the context of my paper. I was implying that, where gear couplings are used on boiler feed pumps in CEGB power stations, site balancing has not generally been found necessary. It was not intended as a completely universal statement. However, the need for correcting balance after erection is a matter of coupling weight, natural frequencies of overhangs, which will be affected by distance of pivot planes from bearing centres, and effect of any inaccuracies in manufacture. With accurate machining and using the arrangement (a) of Fig. 1 of Paper B1 these factors are generally favourable for gear couplings; these combined with the fact that couplings on feed pump drives never run unloaded so that centering due to transmitted torque will always take place are probably the explanation for site balancing being unnecessary. In my understanding at high speeds diametral location is unnecessary. Torque loading – the Conti-Barbaran effect (Reference 3 of my Paper) – gives the required centering. I do not know the design clearances between the teeth. This is for the gear coupling manufacturers or, possible, Michael Neale and Associates to comment.

With high speed grease lubricated gear couplings – 4,000 – 5,000 rpm – my information is that the grease escaped by penetrating the joints.

CEGB do not "favour" straight teeth. With the sizes of couplings described in my Paper running at the speeds considered, the permissible sliding speed of 125 mm/sec limits the allowable inclination to 1 to 2 in 1000 (3' –6'). With such small inclinations the gain from crowning and barreling grossly in excess of the misalignments that they can carry when permissible sliding velocities are considered. A small amount is not wrong on high speed couplings but in many applications that benefit may be small or even non-existent. Regarding the Cottam failures described in my Paper it is just possible that high pressures at the teeth end may have been a factor, but many of the total of 180,000 hours run with straight teeth have taken place since the failures occurred in 1970 but before the opportunity to add crowning and barreling became available. The very similar pumps and gearboxes at Rugeley 'B' have retained straight teeth. Further, the comment by Mr. J. Wright at the bottom of page 3 of his Paper, B4, implies that for high speed and high power couplings the radius desired is so large that effectively the teeth will be straight.

It is probable that Boylan's experience (Reference 1 of my Paper) was entirely with crowned and barreled teeth. Nonetheless his criterion appears to be a suitable one for straight teeth too.

Mr. Boole gives tribute to the high standards of alignment procedures of the CEGB; these are really those set by the present manufacturers. My impression is that they are not exclusive to us but are the normal standards required for high power and high speed machines.

Y.N. CHEN (Sulzer Brothers Limited, Switzerland)

Mr. Otway is to be congratulated for giving a very useful criterion for the suppression of instability in the gear coupling spacer of the centrifugal pump units. Since the case treated by the writer in 1973 was cited in the paper (Ref. 2 page F2-19), some more details of his measurement are given below, as a further contribution to this interesting paper.

Fig. FD1 shows the vibration behaviour of the pump rotor and the HS gear coupling spacer of the pump unit, consisting of driving motor, LS gear coupling, gear box, HS gear coupling and the pump (HS= high speed side, LS = low speed side), during the running up procedure at leak-off operation condition from  $n_p = 74$  to III RPS with the original gear coupling as shown in the upper part of Fig. 4 in the paper. All the amplitudes having the pump running speed frequency show a very strong beat whose frequency obeys a linear law with the running speed of the pump  $n_p$ , namely

$$f_{\text{beat}} = 0.162n_p,$$

as shown in the lower subfigure. The amplitude of the beat is considerably larger at the coupling spacer than at the pump rotor in the whole speed range. The origin of the excitation of the beat must therefore be searched for in the system of the gear coupling. The coupling must perform a movement causing an excitation force of the frequency of  $(1 - 0.162) n_p = 0.383 n_p$  in order to produce this beat.

The main vibrations of the pump bearing on the gear box side, the gear box bearings on the HS coupling and motor sides are compiled in Fig. FD2. The vibration of the pump bearing does not show the pump running speed frequency, but only a constant frequency of 190 Hz in the entire speed range. In addition, a vibration corresponding to the double slip frequency of the motor appears primarily in the horizontal direction of these three bearing. This vibration is not reproduced here because of its irrelevancy to the present problem.

The vibrations of the gear box bearings exhibit primarily a constant frequency of  $f = 68$  Hz, corresponding to the natural frequency of the gear box casing. They have a transient behaviour in the lower speed range, changing over to a steady behaviour when reaching the maximum speed of  $n_p = \text{III RPS}$ . The component with the pump running speed frequency  $n_p$  is very small. It is thus obvious that

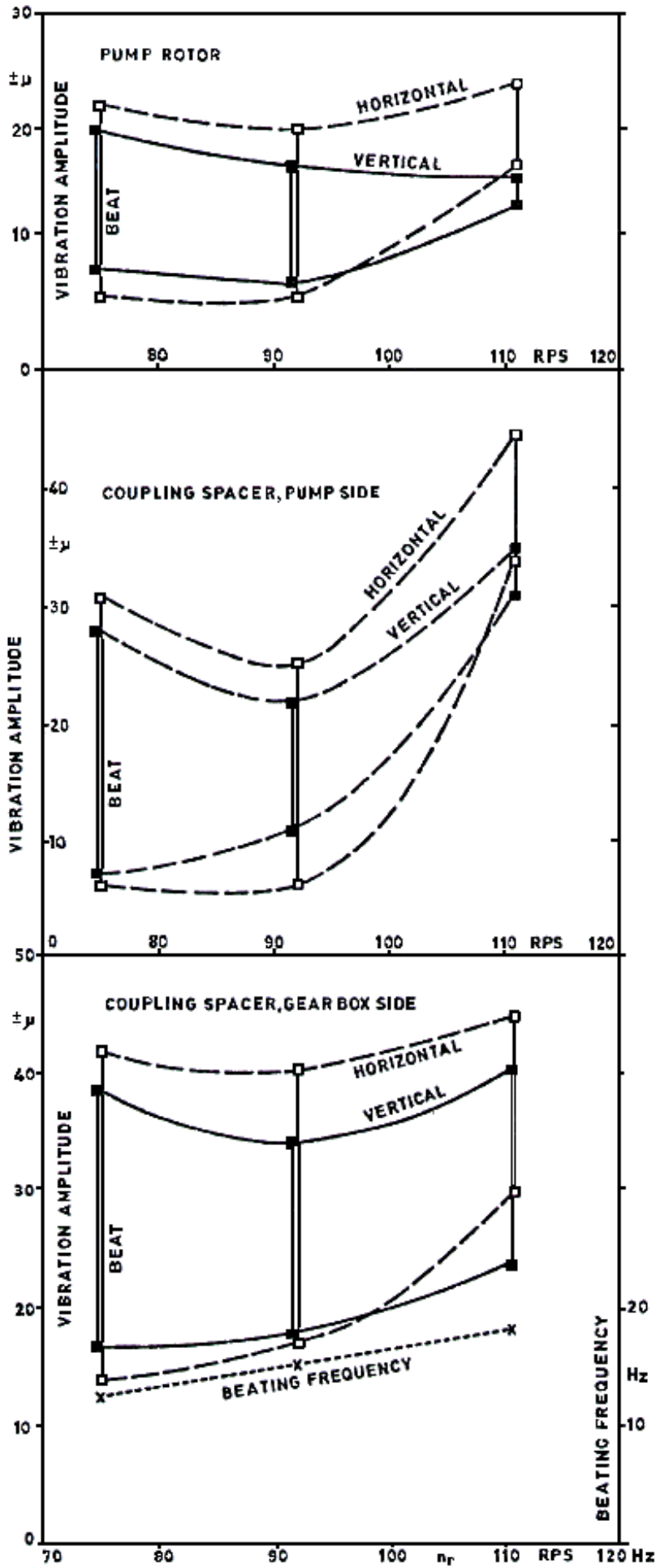


FIG. FD1



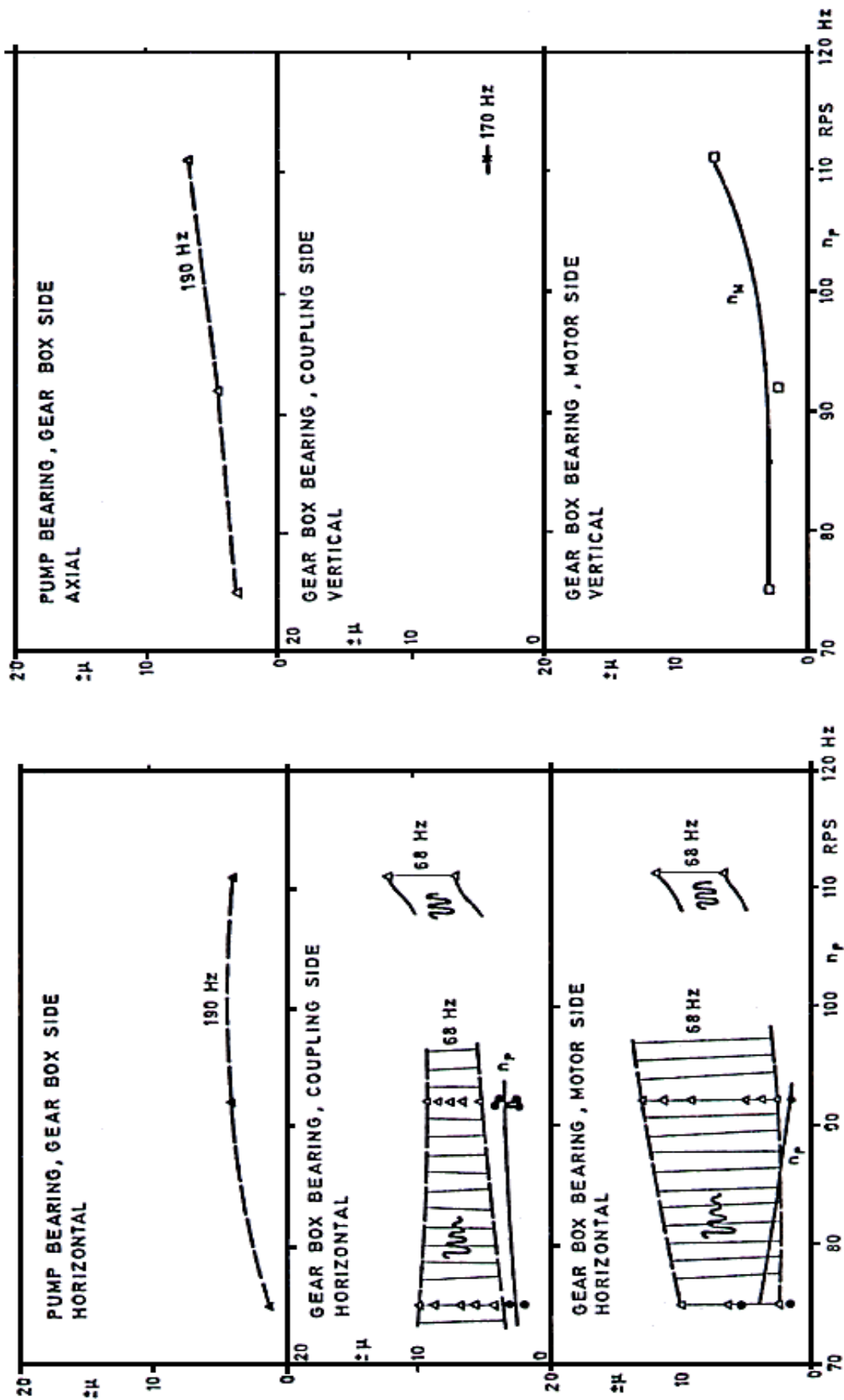


FIG. FD2

the natural vibration of the gear box is excited by impulsive forces in the system in a random manner in the lower speed range. The steady behaviour of the vibration at  $n_p = 111$  RPS, however, must be attributed to a steady excitation. The difference tone between the pump speed frequency, the beat frequency and the motor speed frequency according to the following equation

$$f = n_p - f_{\text{beat}} - n_M \quad (n_M = 0.217n_p) \quad (2)$$

will possess a frequency of 68 Hz at  $n_p = 110$  RPS. It can thus be argued that this difference tone is the source of the excitation. As the gear box is directly connected to the HS gear coupling and the motor shaft, the excitation must be originated from this gear coupling and the motor. The above equation can therefore be converted into the following form:

$$f = n_{\text{coupling}} - n_M \quad (3)$$

$$\begin{aligned} \text{where } n_{\text{coupling}} &= n_p - f_{\text{beat}} \\ &= (1-0.162) n_p \\ &= 0.838 n_p \end{aligned}$$

In this manner, the vibration of the gear box can be found to originate from the same excitation mechanism as the beat of the gear coupling. This means that it is the same movement of the HS gear coupling which excites the beat of the rotor system in the entire speed range, and the steady vibration of the 68 Hz of the gear box at the maximum running speed. This movement was described to be the free orbital movement of the coupling spacer in the previous paper of the writer (Ref. 2 page F2-19). The parameter  $\epsilon$  given by Conti-Barbaran (Ref. 3 page F2-19) seems only to supply a criterion for the static centering of the gear coupling, and not for a dynamic phenomenon such as the gyroscopic instability of the gear spacer.

From the curves for the pump rotor vibration in Fig. FD1 it is obvious that no critical speed immediately above the maximum rated speed  $n_p = 111$  RPS can be derived. The mean shapes of the curves for both the horizontal and vertical directions rise only slightly up to this speed. The critical speed must therefore lie far beyond the operating speed range. The fact that no running speed frequency at all was detected at the gear-box-side pump bearing in the entire speed range, supplied further support to this conclusion. The critical speed of about 7000 RPM = 117 RPS, as supposed in the paper, is thus not confirmed. Only when the gear coupling was replaced by the heavy flexible coupling, the critical speed seems to be lowered to approach the maximum running speed.

The benefit of the thickening of the pump shaft and the shortening of its overhang, as shown in the lower half of Fig. 4, appears to lie in the reduction of the deflection angle of the coupling spacer on the pump side. The initiation of the gyroscopic movement of the spacer is thus eliminated. The writer would like to suggest an investigation to determine whether a short overhang and a light coupling spacer would already enable the orbital instability of the gear coupling to be suppressed without requiring the natural frequency of the pump shaft end to have a value as high as 50% above the running speed. Then, a less thick pump shaft would suffice for smooth operation of the gear coupling.

AUTHOR'S REPLY : F.O.J. OTWAY (C.E.G.B., England)

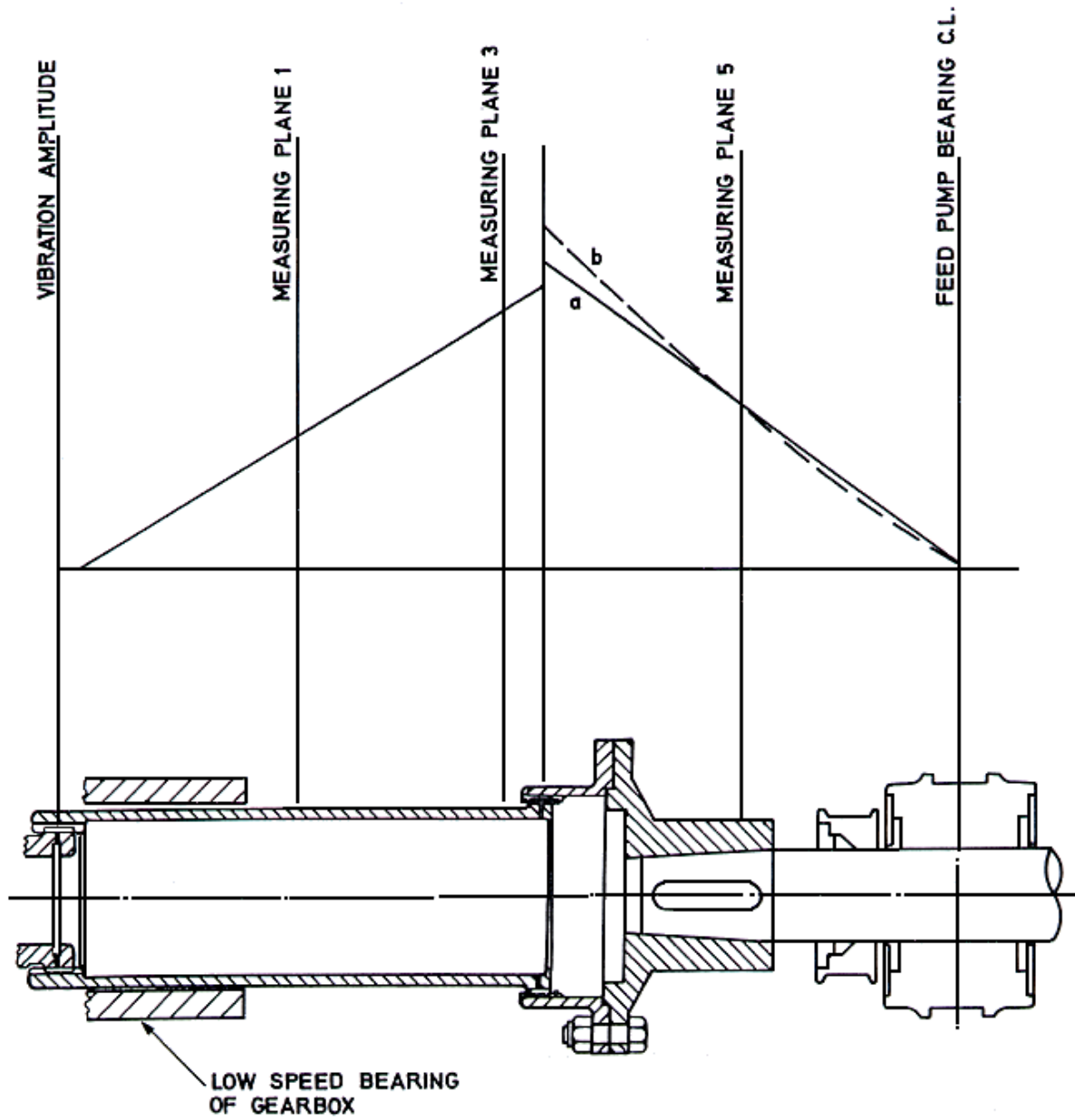
I am grateful for Dr. Chen's contribution. Referring to Fig. FD1 of his paper it is correct that the coupling spacer at the pump end showed the greatest amplitude. The situation was as set out in Fig. FD3, which shows – approximately to scale – the measuring planes and the amplitude will occur in measuring plane 3 and the fact that it is the greatest gives no indication as to the cause of the vibrations. In Fig. FD3 the extrapolation to the plane of the coupling teeth adjoining the pump has been taken as straight since the spacer will have been rigid. The pump shaft is more flexible; accordingly the extrapolation would probably have been more correct as the broken curve 'b' rather than the straight line 'a'.

The question of frequencies is more complex. The frequency of  $0.162 n_p$ , mentioned by Dr. Chen, did not arise in the coupling spacer but in the gearbox, where manufacturing errors, which are extremely small, can produce a detectable but very small movement of the sunwheel at this frequency.

The gearbox bearing did show a frequency of 68 Hz. I do not consider that the explanation is related to motor slip but rather to the natural frequency of the gearbox. Dr. Chen refers to possible gyroscopic instability of the coupling spacer. At the beat frequency of  $0.162 V$  and amplitude of  $0.05 \text{ mm}$  ( $0.002''$ ) the gyroscopic couple is less than  $1.3 \text{ Nm}$  ( $1 \text{ lb. Ft.}$ ). Accordingly it can be ignored.

Both pump rotor and coupling spacer showed increase in amplitude as pump speed increased. The relative magnitudes have been explained in Fig. FD3. The evidence suggested that the pump speed could have been close to a critical vibration frequency of the shaft overhang. It was noted that the amplitude was influenced by the angular position of the coupling flange bolts, to which weights were added deliberately to determine the effects of unbalance. It was suggested that the source of instability was in the coupling spacer. If there were no effect on that part of the coupling attached to the pump shaft due to the addition of weight to a coupling bolt, then, since nothing had been changed on the coupling spacer, no difference in behaviour would have occurred to the spacer. Accordingly our deduction was that the source of the problem was in the pump shaft. When site balancing was carried out on the rotating assembly, correction was only possible in one plane – that of the coupling bolts. It is therefore probable that any correction made in site balancing was not in the correct plane. As a result, although the best achievable balance was made, it was very likely that there were residual couples in the pump shaft. If so, adding weights in a position that would have increased the couple could have led to instability, particularly if close to a shaft natural frequency, whereas adding weights in the diametrically opposite position would have given a stable situation.

I am still of the opinion that the benefit of the thicker shaft lay in increasing the frequency of the pump shaft overhang. As mentioned previously, gyroscopic effects were not large enough to have had any influence. Clearly more investigations could be carried out. However, the pumps have now been running very satisfactorily for a number of years in their present form – the lower arrangement of Fig. 4 of my Paper. There is not sufficient justification for disturbing plant that is in operational service



**TILBURY B STARTING / STANDBY FEED PUMPS  
VIBRATION AMPLITUDES AND POSITION OF  
MEASURING PLANES DURING TESTS**

**FIG. FD3**

A.W. LEES (C.E.G.B.), England

Torsional oscillations in rotating plant have been a source of trouble in several instances. This fact has usually been allowed for in new specifications by stipulating that torsional critical speeds should be removed from the operating range; and clearly the choice of flexible couplings, where appropriate, has a considerable effect on these critical speeds. A more detailed study of the forces arising in a geared system, however, shows that the appropriate choice of critical speeds is not the sole criterion for good dynamic performance.

Any gearbox will inevitably have some manufacturing errors. Ignoring flexural vibrations within the gearbox, these errors will give rise to a form of displacement – controlled torsional vibration. A full account of the forces arising has recently been discussed by Lees and Haines (1). A very much simplified analysis is given here with the following notation:-

K	Stiffness
$l_1$ ) )	Moments of inertia of pinion and wheel
$l_2$ )	
P	Torque on pinion
$R_1$ ) )	Radii of wheel and pinion
$R_2$ )	
$\sigma_1, \sigma_2, \sigma^1$	Damping terms
$\gamma$	Gear ratio
$\omega$	Angular speed
$\omega_c$	Critical speed
i	$\sqrt{-1}$

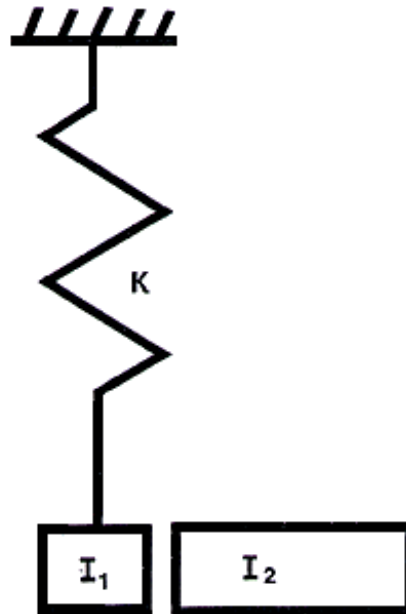
Consider a system in which an infinitely massive prime mover (i.e. a turbine) is connected via a coupling of torsional stiffness K, through a gearbox, to a brake, which is rigidly coupled and has negligible inertia. The system is shown in Fig. FD4.

Let  $\theta_1$  and  $\theta_2$  be the angular deflections of the pinion and wheel respectively.

$$\text{Then } \theta_1 = \frac{P}{K - l_1 \omega^2 + i \omega \sigma_1} \quad (1)$$

$$\text{and } \theta_2 = \frac{\gamma P}{l_2 \omega^2 - i \omega \sigma_2} \quad (2)$$

### SIMPLE MODEL



**FIG. FD4**

These angular displacements may take any values, but are interrelated in that their weight sum (not their difference since wheel and pinion move in opposite senses) must be equal to the gear pitch error, i.e.

$$R_1 \theta_1 + R_2 \theta_2 = e \quad (3)$$

$$\frac{P}{(K - I_1 \omega^2 + i\omega\sigma_1)} - \frac{\gamma^2 P}{(I_2 \omega^2 - i\omega\sigma_2)} = \frac{e}{R_1} \quad (4)$$

$$P = \frac{e}{R_1} \frac{(K - I_1 \omega^2 + i\omega\sigma_1) (I_2 \omega^2 - i\omega\sigma_2)}{-I_2 \omega^2 + i\omega\sigma_2 + \gamma^2 (K - I_1 \omega^2 + i\omega\sigma_1)} \quad (5)$$

Clearly this system has a single resonance at frequency  $\omega_c$  given by

$$\omega_c^2 = \frac{\gamma^2 K}{\gamma^2 I_1 + I_2} \quad (6)$$

and the above expression will reduce (for small damping) to

$$P = \frac{e}{R_1} \frac{\left[ 1 - \frac{\gamma^2 I_1}{\gamma^2 I_1 + I_2} \left( \frac{\omega}{\omega_c} \right)^2 \right] I_2 \omega^2}{\gamma^2 (1 - \omega^2 / \omega_c^2) + \frac{i\sigma_1 \omega}{K} \gamma^2} \quad (7)$$

where  $\sigma^1$  is a composite damping term

$$\sigma^1 = \gamma^2 \sigma_1 + \sigma_2$$

The situation described is constrained to have finite displacement and so when

$\omega \gg \omega_c$

$$P = \frac{e}{R_1} \frac{I_1 I_2 \omega^2}{I_2 + \gamma^2 I_1} \quad (8)$$

at the opposite limit  $\omega \ll \omega_c$

$$P = \frac{e}{R_1} \frac{I_2 \omega_c^2}{\gamma^2}$$

Whilst it is very important to know the torsional critical speeds of a complete machine, this knowledge is only a means to an end. The parameter of real physical importance is the force between the gear teeth. Consider now the torque amplitudes occurring at the critical speeds of our simple model. Putting  $\omega = \omega_c$  gives

$$\begin{aligned} P &= \frac{K e}{R_1} \frac{I_2^2 \omega_c^2}{(\gamma^2 I_1 + I_2) \sigma^1 \omega_c \gamma^2} \\ &= \frac{e}{R_1} \frac{I_2^2 \omega_c^3}{\gamma^4 \sigma^1} \end{aligned} \quad (9)$$

The situation is displacement-controlled and is in marked contrast to the force-controlled vibration resulting from out-of-balance on a rotor. In that case the displacement amplitude tends to zero as the speed tends to infinity. (See for example Den Hartog (1956).

If an external torque  $Q$  is applied to the wheel at angular frequency  $\omega$ , the resulting torque on the pinion teeth is given by

$$P = \frac{Q \left[ 1 - \frac{I_1}{I_2 + \gamma^2 I_1} \left( \frac{\omega}{\omega_c} \right)^2 \right]}{\gamma \left[ \left( 1 - \left( \frac{\omega}{\omega_c} \right)^2 \right) + \frac{i \omega \sigma^1}{K} \right]} \quad (10)$$

Details of the three cases studied are shown in Table 1. The damping is assumed to give a dynamic magnification of 10 in the spring

$$\text{i.e.} \quad \frac{\sigma^1 \omega}{K} = 0.1$$

	$K_{Nm/rad}$		$I_1 \text{ Kgm}^2$	$I_2 \text{ Kgm}^2$	$c/2$	Hz
A	$10^5$	2	0.05	0.8	100	
B	$2.10^4$	2	0.01	0.16	100	
C	$1.125.10^6$	2	0.25	4.0	150	

**TABLE 1 PARAMETERS OF CASES STUDIED**

Figure FD5 shows a curve for each of the three cases with a gear pitch error. It should be noted that the largest torque oscillations arise (at 100 Hz) in case C, even though the critical speed is well removed from the running range.

In contrast, the response of a gearbox to a fixed external torque is shown. Comparing A with C, it is seen that the response is reduced in Case C although the torque fluctuations due to an error increased. Hence, the torque arising in the displacement controlled situation follows a different curve to a force controlled process.

The analysis of a more complex system is given in Reference 1, in which it was shown how the torque variations of a real system may be calculated using the finite-element method.

From the foregoing point, it can be concluded that the torsional oscillations of a geared system due to gearbox errors depend on system properties. The requirement that torsional critical speeds be removed from the running range is insufficient to ensure good performance in respect of dynamic tooth loading.

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## REFERENCES

1. Lees, A.W. and Haines, K>A>, "Torsional Vibrations of a Boiler Feed Pump" ASME Paper No. 77-DET-28 for ASME, Vibration Conference, Chicago, September 1977.
2. Den Hartog, J.P "Mechanical Vibrations" McGraw Hill, 4th Ed. 1956.



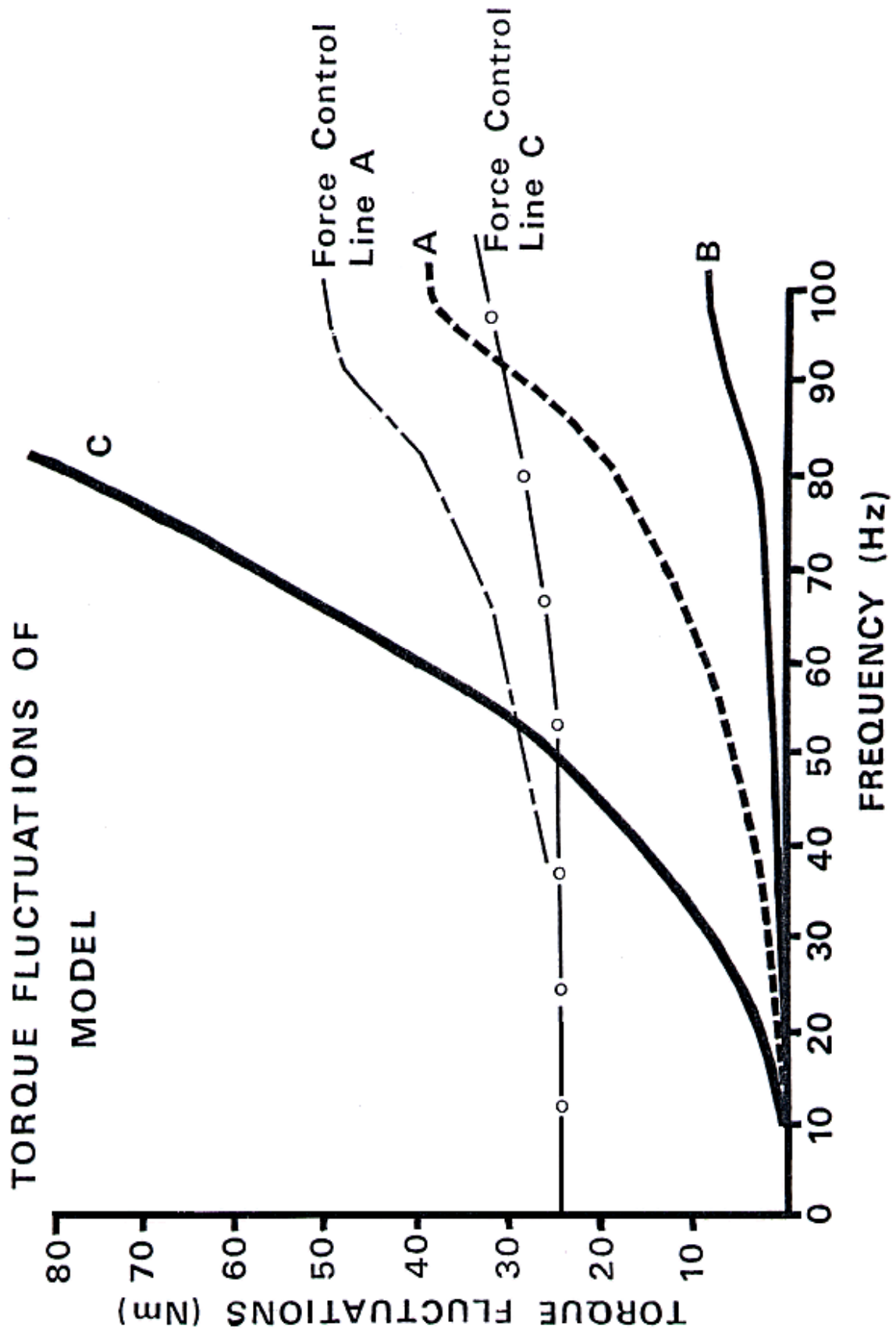


FIG. FD5

AUTHOR'S REPLY : F.O.J. OTWAY (C.E.G.B., England)

DR. lees argues that the torsional oscillations of a geared system due to gear box errors depend on system properties and that it is an insufficient requirement to ensure good performance that the torsional critical speeds be removed from the running range.

If gear pitch errors are capable of inducing torsional vibrations when the torsional natural frequencies are remote from the running speed – which he appears to be arguing – it is necessary to explain why many boiler feed pump drives have not suffered in this way. There is no reason to believe that the Cottam Power Stations was significantly different from those supplied by the same manufacturers to other CEGB Power Stations where no torsional vibrations have been experienced.

The reference in my paper to the Rugeley 'B' and Cottam main boiler feed pumps was to show that coupling failure can be induced by running on or close to natural torsional vibrations. In one case, that of Rugeley 'B', removal of the torsional natural frequencies from the running range has improved the performance so that coupling failures is unlikely.